1965

Resource utilization and productivity estimates in the food and kindred products industry

Paul Douglas Doak
Iowa State University

DOAK, Paul Douglas, 1929-
RESOURCE UTILIZATION AND PRODUCTIVITY ESTIMATES IN THE FOOD AND KINDRED PRODUCTS INDUSTRY.

Iowa State University of Science and Technology
Ph.D., 1965
Economics, agricultural

RESOURCE UTILIZATION AND PRODUCTIVITY ESTIMATES IN THE FOOD AND KINDRED PRODUCTS INDUSTRY

by

Paul Douglas Doak

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1965

TABLE OF CONTENTS

INTRODUCTION

The welfare of consumers and agricultural producers depends to a large extent upon the efficiency, both economic and technological, of the marketing system that is responsible for moving products from the producer to the consumer. An important part of this system is the food processing industries and it is becoming more important as processors perform more and more services for the consumer.

The Problem

The problem in this study is primarily fourfold: 1. to estimate the elasticity of substitution of labor for capital in the food processing industries; 2. to estimate the technological change or shift in the production function of the food processing industries; 3. to examine the behavior of the relative factor shares over time; 4. to examine the relationship, if any, between the rate of return on total assets and the estimates of technological change.

Relevancy of the study

It is commonly recognized that competitive forces, where they exist, will pass on to the consumer and back to the producer cost reductions that are brought about in the marketing channels. Certain actions by government agencies can affect volume and cost variables and thereby affect both producers and consumers.

Government policies can force change if deemed desirable and, just as importantly, can alleviate undesirable effects of change occurring from natural market forces. If, however, a policy is implemented to change or alleviate an undesirable situation, it is necessary for the policy makers to have information concerning the parameters of the production function of the industry involved.

It is anticipated that the coefficients estimated in this study will be useful in later studies involving the relationships between the production function, and structure, conduct and performance variables of markets. Also, if this study is successful the analysis will be extended to other segments of the marketing channels. Finally, the results obtained should be of some considerable importance to economic theory regarding the issue of the constancy of factor shares.

Review of Literature

Douglas (1) was one of the earliest research workers to estimate an aggregate production function using statistical methods. Relying on the help of his associate, Charles W. Cobb, he estimated the aggregate production function to be adequately described by the following:

$$P' = bL^{k}C^{1-k} \qquad (1)$$

where P' is estimated production, b is a constant, L is labor and C is fixed capital.* They estimated b to have a value of 1.01 and k to be equal to .75. A later study for Massachusetts for the period 1890-1926 produced similar results. Later studies using cross-section methods gave values of about .65 for k. The implications of the studies by Douglas and his associates were: 1. the aggregate production function was linear and homogeneous; 2. the relative shares of capital and labor were constant over time; 3. the elasticity of substitution of labor for capital was unity regardless of the capital-labor ratio.

The principle objections to the conclusions drawn by Douglas and his associates from their time series studies were: 1. Mendershausen (2) pointed out that the observations on production, labor and capital constituted a multicollinear set with each variable a function of time; 2. Brown (3) objected to the Cobb-Douglas function being called a production function when applied to time series data as it merely described the historical rates of growth of labor, capital and output.

The relationships between labor, capital, and production

---

*The notation used here is the same as used by Douglas, whereas different symbols are used to denote the same variables or parameters in other sections.

were estimated by Mendershausen to be

$$\log P = .0156\ t \tag{2}$$

$$\log L = .0112\ t \tag{3}$$

$$\log C = .0281\ t \tag{4}$$

and he demonstrated that k was the ratio of the difference in the trend slopes of production and capital, and labor and capital.

Douglas' (4) reply to his critics was that the Cobb-Douglas function had given such consistent results in so many studies in so many different countries that it must be useful for studying the relationships between aggregate output and inputs. The use of the Cobb-Douglas function in aggregate studies eventually declined but it was used more and more at the firm or plant level.

The constant proportions or fixed input function was revived and made popular by Leontief (5) and it was, and continues to be, widely used in aggregate analysis for planning purposes by government agencies.

The use of either the Cobb-Douglas or the constant proportions production functions, with constant returns to scale, are rather restrictive as the elasticity of substitution of labor for capital is either one or zero. The Cobb-Douglas and the Leontief functions are mentioned here because they have played such an important role in production theory. They will also play, indirectly, an important role in the model used in this study. A production function which includes the Cobb-Douglas and Leontief functions as special cases will be presented in the next chapter.

Technological change

Technological change can be defined as any kind of shift in the production function. An advance in technology could be caused by discovery of new machines, new materials, or improved skills and know-how of the human resources.

The term production function, in its broadest interpretation, is a description of output in response to all factors of production whether the factors are in widespread use or not. The term technology refers to the various methods, inputs and materials with which a product can be produced. At any given time, there will be some methods and inputs that would be used if their relative prices were more favorable. Hence, the term current techniques should be used to distinguish between what is possible and what is observed. The term technological change as used later will actually refer to change in current techniques.

Technological change can be visualized easiest by referring to a set of iso-quants with all iso-quants representing the same output. Then as technological advance occurs the iso-quants move closer to the origin with the same output requiring less of one or all inputs. This is portrayed in Figure 1 by the dated iso-quants where z_i represents the time period.

Neutral technological change Salter (6) defines technological change as being neutral if the factor proportions are unchanged when their relative prices are constant at the different time periods. Solow (7) defines neutral technological advance as pure scale changes which leave marginal rates of substitution unchanged at given capital-labor ratios. Since Salter requires relative prices to remain constant and Solow requires the marginal rates of substitution to remain constant, and because price ratios and substitution rates must be equal for the optimum combination of inputs, the two definitions are equivalent. Hicks (8) defines neutral technological advance as being when the ratio of marginal products are unchanged with a constant capital-labor ratio. This definition also seems to be equivalent to Salter's and Solow's. However, Hicks defines capital- and labor-saving changes in terms of changes in marginal products with the capital-labor ratio being invariant because his model is concerned with aggregative analysis with the supplies of capital and labor being given. Harrod (9) defines neutral technological advance as that where the productivity of capital remains constant with the rate of interest being invariant. His analysis, like Hicks', is concerned with aggregative analysis. Following the

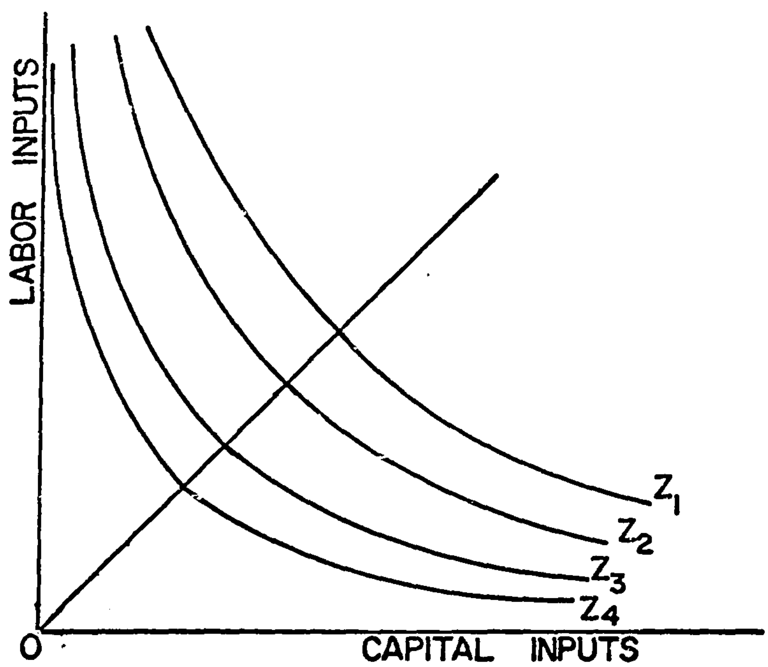

Figure 1. Iso-quants illustrating technological change

definition of Solow and Salter, neutral technological advance can be represented by a straight line from the origin which passes through the equilibrium points at each time period.

Factor-saving technological change The factor savings brought about by technological advance can be measured by the relative change in capital per labor unit when relative factor prices are constant. Since only in the extreme case of zero elasticity of substitution could one factor be increased without increasing the input of the other, while holding output constant, technological change results in relative declines of usage of one factor compared to the other factor. This relative change can be measured by an index of change and in the Lasyperes form this can be represented by

$$T = \left[K_{z_1} / L_{z_1}\right] / \left[K_{z_2} / L_{z_2}\right] \tag{5}$$

where K and L refer to the quantities of the two factors in use at the different time periods and is illustrated in Figure 2. The points E_1 and E_2 represent the optimum combination of factors to use when prices of period one are used. The price-ratio lines showing the relative prices of period one are illustrated by lines P_1 and P_2. The proportionate change in capital is greater than the proportionate change in labor, as T is greater than one, and the example

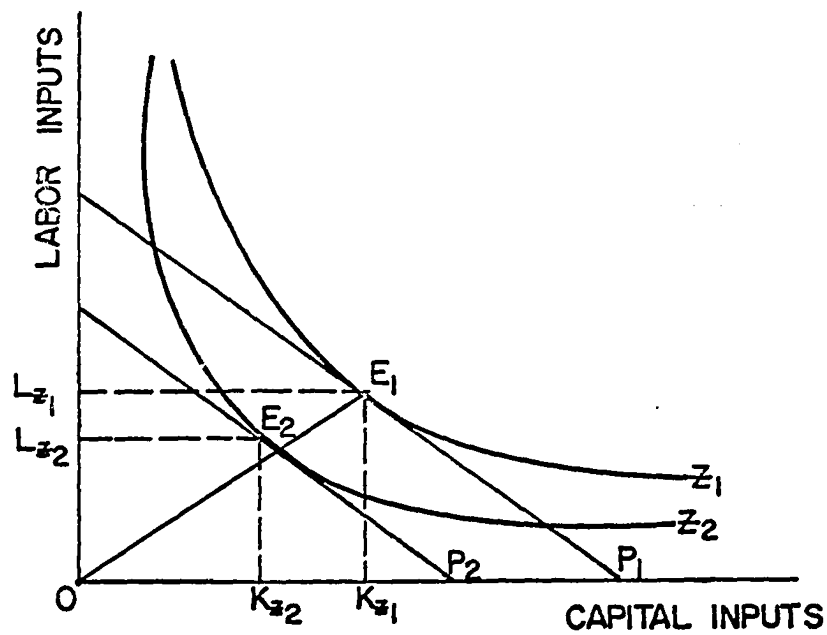

Figure 2. Illustration of factor-saving bias

has a capital-saving bias. If the point E_2 fell on the line OE_1, T would equal one, and technological change would be neutral. If E_2 fell to the right of OE_1, T would be less than one, and the technological advance would have a labor-saving bias.

<u>Technological change and the elasticity of substitution</u> Advances in technology not only shift iso-quants but can also alter the relative proportions of factors and change the shapes of iso-quants. The curvature of the iso-quants reflects the elasticity of substitution and hence it can also be affected. Salter (6) suggests that advances which are only applicable over a small range of the iso-quant, such as an improvement in an existing machine, reduce the elasticity while advances such as electronic computers and control devices increase the elasticity of substitution. Also, technical advances which increase or decrease the elasticity of substitution lead to an acceleration of the rate of productivity growth of one factor and a slowing down of the rate of productivity growth of the other. If labor is becoming higher priced relative to capital, and the elasticity is increasing, the rate of growth of labor productivity will be going up while that of capital productivity will be slowing down.

<u>Measures of technological change</u> Salter (6) suggests measuring technical advance between time periods by the

relative change in total unit costs when the techniques used in each period would minimize unit production costs with factor prices being held constant. A method of measuring technological advance developed by Solow (7), which will be discussed later, was used by Massel (10) to estimate technological change in manufacturing in the United States between 1919 and 1955. He estimated the technological change parameter to have changed from an arbitrary figure of one in 1919 to nearly three in 1955. That is to say, if output per man-hour changed from $1.00 per hour in 1919 to $9.00 in 1955 that $9.00 ÷ 3.00, or about $3.00, represents what output would have been without any technological change. Hence, $6.00 increase in output must be due to technological change and $2.00 increase must be due to increased capital in use per man-hour.

THE MODEL

The model used in this analysis is based on the theory of the firm. Although this study relies heavily on the purely competitive system it does not depend entirely upon it as the firms involved can be monopolistic competitors in the product markets. The firm is defined as a profit maximizing, decision making unit. The firm must make decisions concerning the optimum combination of factors of production to use, the optimum combination of products to produce, and the optimum level of output of a product.

The Optimum Combination of Factors

The firm will produce any given output as cheaply as possible or conversely maximize output from any given expenditure. The combination of inputs selected depends upon the productivity of the factors in conjunction with the relative prices of those factors. This is easily demonstrated if we assume that the firm produces only one product.

Let the technical conditions of production be represented by

$$Q = f(X_1, X_2, \ldots, X_n) \qquad (6)$$

where Q denotes the quantity of the product and the X_is are the factors of production. Let the cost function be represented by

$$C = P_1X_1 + P_2X_2 + \ldots + P_nX_n \tag{7}$$

where C represents total cost and the P_is represent the cost of each factor. The objective is to maximize Equation 6 subject to the given expenditure represented by Equation 7. In order to achieve this objective the following constrained function is formed:

$$Q = f(X_1, X_2, \ldots, X_n) - \lambda(P_1X_1 + P_2X_2 + \ldots + P_nX_n - C) \tag{8}$$

where λ is the Lagrangean multiplier (11). The necessary condition for maximizing output is that the partial derivatives of Q with respect to the X_is are equal to zero.

$$\begin{aligned}
\frac{\partial Q}{\partial X_1} &= \frac{\partial f}{\partial X_1} - \lambda P_1 = 0 \\
\frac{\partial Q}{\partial X_2} &= \frac{\partial f}{\partial X_2} - \lambda P_2 = 0 \\
&\vdots \\
\frac{\partial Q}{\partial X_n} &= \frac{\partial f}{\partial X_n} - \lambda P_n = 0 \\
\frac{\partial Q}{\partial \lambda} &= -(P_1X_1 + P_2X_2 + \ldots + P_nX_n - C) = 0
\end{aligned} \tag{9}$$

$$(i = 1, \ldots, n)$$

where $\frac{\partial f}{\partial X_i} = MPP_i$

represents the marginal physical product of the ith factor. The necessary condition for the optimum combination of factors is given by

$$\frac{MPP_1}{P_1} = \frac{MPP_2}{P_2} = \dots = \frac{MPP_n}{P_n} \tag{10}$$

The sufficient condition for the solution shown by Equation 9 to be a maximum is that $d^2Q < 0$ for any variation when $dQ = 0$.

The Optimum Level of Output of a Product

If the firm produces only one product the intensity of factor use can be determined by maximizing profit with respect to each input. The profit equation is expressed by the following:

$$R = P_q Q - (P_1X_1 + P_2X_2 + \dots + P_nX_n) \tag{11}$$

where R equals net profit and P_q equals the price of the output. When Equation 11 is maximized with respect to each X_i the following is obtained:

$$\frac{\partial R}{\partial X_1} = P_q \frac{\partial Q}{\partial X_1} - P_1 = 0 \tag{12}$$

$$\frac{\partial R}{\partial X_2} = P_q \frac{\partial Q}{\partial X_2} - P_2 = 0$$

$$\vdots$$

$$\frac{\partial R}{\partial X_n} = P_q \frac{\partial Q}{\partial X_n} - P_n = 0$$

The necessary condition for optimum output of the single product is given when

$$MPP_i = \frac{P_i}{P_q} \tag{13}$$

The sufficient condition is given by $d^2R < 0$ for any variation when $dR = 0$. It is readily apparent from Equation 13 that the optimum usage of any factor is equivalent to the optimum output of a product.

The Optimum Combination of Products

The determination of the optimum combination of products is more difficult than choosing the optimum combination of factors. This is because the technical conditions of production for each product must be considered simultaneously with the prices of the products. In addition, some products may be factors used in the production of other products.

Allen (11) suggests that all factors and products be considered as products with factors being denoted as negative products. The technical conditions of production can then be denoted by

$$f(X_1, X_2, \ldots, X_r, X_{r+1}, X_{r+2}, \ldots, X_{r+s}) = 0 \qquad (14)$$

If $i \leq r$, X_i is negative and is an input, if $i \geq r + 1$, X_i is positive and is an output. Equation 14 can be simplified further to

$$f(X_1, X_2, \ldots, X_n) = 0 \qquad (n = r + s) \qquad (15)$$

The profit equation is expressed as a function of costs and revenues and can be written as

$$R = \sum_{i=1}^{n} P_i X_i \; . \qquad (i = 1, \ldots, n) \qquad (16)$$

The optimal solution is obtained by maximizing the profit function, subject to the technical conditions, with respect to each X_i.

$$R = \sum_{i=1}^{n} P_i X_i - \lambda f(X_1, X_2, \ldots, X_n) \qquad (17)$$

$$\frac{\partial R}{\partial X_i} = P_i - \lambda f_i = 0 \qquad (18)$$

$$\frac{\partial R}{\partial \lambda} = -f(X_1, X_2, \ldots, X_n) \qquad (19)$$

where $f_i = \frac{\partial f}{\partial X_i}$

From Equation 18

$$P_i = \lambda f_i \qquad (i = 1, \ldots, n) \tag{20}$$

$$P_j = \lambda f_j \qquad (j = 1, \ldots, n) \tag{21}$$

and hence

$$\frac{P_i}{P_j} = \frac{f_i}{f_j} \tag{22}$$

If $i \leq r$ and $j \geq r + 1$, or $j \leq r$ and $i \geq r + 1$, Equation 22 expresses the relationship between a factor and a product. Hence, under the stated conditions Equation 22 can be rewritten as

$$MPP_i = \frac{P_i}{P_j} \tag{23}$$

or

$$MPP_j = \frac{P_j}{P_i} \tag{24}$$

which is the same as Equation 13 except for the change in notation of the numerator.

If $i, j \leq r$, then X_i and X_j are both inputs and it becomes apparent that Equation 22 can be rewritten as

$$\frac{P_i}{P_j} = \frac{MPP_i}{MPP_j} \tag{25}$$

or

$$\frac{MPP_i}{P_i} = \frac{MPP_j}{P_j} = \dots = \frac{MPP_r}{P_r} \tag{26}$$

hence Equation 26 gives the optimum combination of factors to use. It is clear that Equations 22 and 26 are identical to Equation 10.

If i, j $\geq$ r + 1 then Equation 22 expresses the relationship between two products and hence X_i and X_j are both products. Equation 22 can be rewritten as

$$\frac{P_i}{P_j} = \frac{\partial X_j}{\partial X_i} \tag{27}$$

or

$$\frac{P_1}{f_1} = \frac{P_2}{f_2} = \dots = \frac{P_n}{f_n} \tag{28}$$

Equation 27 states that the marginal rate of substitution of any two products must be equal to the ratio of their prices in order for the combination of products produced to be an optimum.

The sufficient condition for Equations 24, 26 and 28 to be optimal solutions is that $d^2R < 0$ where $d^2R = -\lambda d^2f$ and $d^2f > 0$ (11).

Price and Quantity Relationships

The nature of the relationship between the change in quantity of any specific X_i due to the change in any price can be investigated by taking the total differential of Equations 18 and 19 and setting them equal to zero. This procedure results in

$$dP_i - \lambda f_{ii}\, dX_i - f_i\, d\lambda = 0 \tag{29}$$

$$f_i\, dX_i = 0 \tag{30}$$

or

$$-\lambda f_{ii}\, dX_i - f_i\, d\lambda = -dP_i \tag{31}$$

$$f_i\, dX_i + 0\, d\lambda = 0 \tag{32}$$

Rewriting Equations 31 and 32 in matrix form results in

$$\begin{bmatrix} -\lambda f_{ii} & -f_i \\ f_i & 0 \end{bmatrix} \begin{bmatrix} dX_i \\ d\lambda \end{bmatrix} = \begin{bmatrix} -dP_i \\ 0 \end{bmatrix} \tag{33}$$

and letting the left-hand member be represented by H and multiplying both sides by H^{-1} produces

$$\begin{bmatrix} dX_i \\ d\lambda \end{bmatrix} = H^{-1} \begin{bmatrix} -dP_i \\ 0 \end{bmatrix} \tag{34}$$

where H^{-1} is the inverse of the matrix H. Let the inverse of H be

$$H^{-1} = \begin{bmatrix} h^{11} & h^{12} & \cdots & h^{1n} \\ \vdots & & & \\ h^{n1} & \cdots & & h^{nn} \end{bmatrix} \tag{35}$$

The values of the h^{ij}s in H^{-1} are defined by the following:

$$h^{ij} = \frac{H^{ij}}{|H|} \tag{36}$$

where the H^{ij}s are the co-factors of the determinant H.

The multiplication indicated in Equation 34 results in

$$dX_i = -h^{i1}\, dP_1 - h^{i2}\, dP_2 - \ldots - h^{in}\, dP_n \tag{37}$$

The relationship between any X_i and P_j can now be determined by finding the partial derivative of X_i with respect to P_j from Equation 37. Hence

$$\frac{\partial X_i}{\partial P_j} = -h^{ij} \tag{38}$$

The following relationships are of interest.

Factor-product relationship

If X_i is a product and P_i is its price, since the sign of H^{ii} and H are opposite, h^{ii} will be positive and the relationship

$$\frac{\partial X_i}{\partial P_i} > 0 \tag{39}$$

signifies that as the price of the *i*th good increases the firm will increase its output.

If X_i is a product and P_j is the price of a factor used in its production then

$$\frac{\partial X_i}{\partial P_j} < 0 \tag{40}$$

indicates that as the price of the *j*th factor increases the output of the *i*th product declines.

If X_i is a factor and P_j is the price of a product which uses X_i then

$$\frac{\partial X_i}{\partial P_j} > 0 \tag{41}$$

and more of the *j*th factor is utilized when P_j increases.

Factor-factor relationships

If X_i is a factor and P_j is the price of the *j*th factor then

$$\frac{\partial X_i}{\partial P_j} > 0 \tag{42}$$

states that the *i*th factor is substituted for the *j*th factor in the production process.

If X_i is a factor and P_i is its price then

$$\frac{\partial X_i}{\partial P_i} < 0 \tag{43}$$

states that less of the *i*th factor is utilized as its price increases.

If X_i is a factor and P_j is the price of a factor then

$$\frac{\partial X_i}{\partial P_j} < 0 \tag{44}$$

indicates that X_i and X_j are complementary factors of production.

Product-product relationships

If X_i is a product and P_j is the price of another product X_j, and

$$\frac{\partial X_i}{\partial P_j} < 0 \tag{45}$$

then X_i and X_j are competitive goods in the production process.

If X_i and X_j are independent or supplementary to each other in the production process, then

$$\frac{\partial X_i}{\partial P_j} = 0 \tag{46}$$

If X_i and X_j are complementary or joint products of the production process, then

$$\frac{\partial X_i}{\partial P_j} > 0 \tag{47}$$

will result.

The Demand for a Factor of Production

The demand for any factor of production depends upon its productivity in producing other factors or finished products. The productivity of a factor of production in money terms is given by

$$MPP_i \; P_j = MVP_i \tag{48}$$

where MPP_i and MVP_i represent the marginal physical product and the marginal value productivity of the *i*th factor and P_j is the price of the *j*th product.

Hence, the demand curve for a factor is identical to the marginal value productivity curve and Equation 48 is identical to Equation 23.

The Returns to a Factor

Equation 43 shows the quantity of factor X_i which a firm will hire or buy to produce product X_j with the prices of each given. Hence, if the owner of a factor also takes prices as given the total wages or rewards are determined by the quantity of factors or services which the owner is willing to sell at that price.

The Elasticity of Substitution

If a product, Q, is produced by the use of two factors of production, X and Y, the production function can be expressed as

$$Q = f(X,Y) \tag{49}$$

Q is constant on the iso-quant in the (X,Y) plane, hence dQ is equal to zero

$$dQ = \frac{\partial Q}{\partial X}\,dX + \frac{\partial Q}{\partial Y}\,dY \tag{50a}$$

Solving for $\frac{dY}{dX}$ results in

$$-\frac{dY}{dX} = \frac{\partial Q}{\partial X} \Big/ \frac{\partial Q}{\partial Y} \tag{50b}$$

or

$$-\frac{dY}{dX} = \frac{MPP_x}{MPP_y} = Z \tag{50c}$$

where Z stands for the marginal rate of substitution of factor X for factor Y in the production of Q. This ratio decreases as more X and less Y is used with the output of Q held constant.

<u>The rate of change of the marginal rate of substitution</u>

The elasticity of substitution of labor for capital is defined by Allen (12) as the percentage change in the capital-labor ratio in response to the percentage change of the slope of the iso-quant. Writing the ratio in proportional terms results in

$$\sigma = \left[\frac{X}{Y} d\left(\frac{Y}{X}\right) \right] \Big/ \frac{dZ}{Z} \tag{51}$$

where the numerator refers to the percentage change in the capital-labor ratio and the denominator refers to the percentage change in the slope.

It will be useful to demonstrate the relationship between the curvature of the iso-quant and the elasticity of substitution. Equation 51 can be rewritten by making the following substitutions:

$$d\left(\frac{Y}{X}\right) = \frac{XdY - YdX}{X^2} \tag{52}$$

and

$$dZ = -\left[Z \frac{\partial Z}{\partial Y} - \frac{\partial Z}{\partial X} \right] dX \tag{53}$$

When the above values are substituted into Equation 51, the elasticity of substitution becomes

$$\sigma = \frac{Z}{XY} \frac{XZ + Y}{Z \frac{\partial Z}{\partial Y} - \frac{\partial Z}{\partial X}} \tag{54}$$

The curvature of the iso-quant at point (X,Y) is given by the total derivative of Z and is shown by

$$\frac{d^2Y}{dX^2} = \frac{d}{dX}\left(\frac{dY}{dX}\right) = -\frac{d}{dX}(Z) = -\left[\frac{\partial Z}{\partial X} - \frac{\partial Z}{\partial Y}\frac{dY}{dX}\right] = Z\frac{\partial Z}{\partial Y} - \frac{\partial Z}{\partial X} \tag{55}$$

However, note that Equation 55 is the same as the denominator of the end term in Equation 54. Hence, the curvature of the iso-quant at (X,Y) is an important element in the elasticity of substitution.

<u>Limiting values of the elasticity of substitution</u>

The limiting values for σ are zero and infinity. If the iso-quant is a straight line intersecting both the Y and X axis then σ equals infinity. If the iso-quant is a right angle convex to the origin then σ equals zero.

Linear Homogeneous Production Functions

A function is defined to be a linear homogeneous function if

$$f(TX,TY) = Tf(X,Y) \tag{56}$$

at any point (X,Y) for any value of T (12). When the variables X and Y in the function

$$Q = f(X,Y) \tag{57}$$

are increased in a fixed proportion the corresponding change in the output may be greater, less than, or exactly equal to the increase in X and Y. If Q always increases in exactly the same proportion as X and Y, the function is said to be homogeneous of the first degree or linear and homogeneous.

The linear homogeneous function is a member of a much wider class of homogeneous functions. The general case is represented by

$$f(TX,TY) = T^{m} f(X,Y) \tag{58}$$

and if this holds true for any point (X,Y) and for any value of T, then Equation 58 is homogeneous of the mth degree.

Euler's theorem

If Equation 58 is homogeneous of degree one, or linear and homogeneous, then the following property holds regardless of the values of X and Y:

$$Q = X \frac{\partial Q}{\partial X} + Y \frac{\partial Q}{\partial Y} \tag{59}$$

Equation 59 is a very important theorem to economics and in particular to the marginal productivity theory of income distribution. If and only if Equation 56 is homogeneous of the first degree at the particular value of (X,Y) will Equation 59 be true. In the language of distribution theory, Equation 59 states that if each factor of production is paid its marginal product the total product will be exactly distributed between X and Y. There will be nothing left over to be claimed by either of the two factors.

Increasing returns to scale

If m is greater than one in Equation 58 output increases at a greater rate than inputs for any positive value of T. This is the case of increasing returns to scale. If the factors are paid their marginal products, the total output is insufficient to pay all factors. Hence, increasing returns to scale is incompatible with factors being paid their marginal products.

Decreasing returns to scale

If m is less than one in Equation 58 output increases at a smaller rate than inputs for any positive value of T. This is the case of decreasing returns to scale. If the factors are paid their marginal products the total output is more than sufficient to pay all factors and there is a surplus to be claimed by a fixed factor or residual claimant.

The Cobb-Douglas function

A specific homogeneous mathematical function which has been widely used in empirical studies by economists is the Cobb-Douglas function. The general form of this function is

$$Q = AX^{\alpha}Y^{\beta} \tag{60a}$$

It is easy to demonstrate the fact that this function is homogeneous of degree m. If

$$f(X,Y) = AX^{\alpha}Y^{\beta} \tag{60b}$$

then

$$f(TX,TY) = A(TX)^{\alpha}(TY)^{\beta} = AT^{\alpha+\beta}\; X^{\alpha}Y^{\beta} \tag{60c}$$

where $\alpha + \beta = m$.

Elasticity of substitution of the Cobb-Douglas function

When the production function is linear and homogeneous, the elasticity of substitution formula becomes of simpler form and can be written as:

$$\sigma = \frac{f_X f_Y}{Q f_{XY}} \tag{61}$$

where f_X and f_Y are the first partial derivatives and f_{XY} is the cross-partial derivative of the production function. Substituting in the appropriate derivative from Equation 60a results in the following situation:

$$\sigma = \frac{A\alpha X^{\alpha-1}\ Y^{\beta}\ A\beta X^{\alpha}\ Y^{\beta-1}}{AX^{\alpha}\ Y^{\beta}\ A\alpha\beta X^{\alpha-1}\ Y^{\beta-1}} = 1 \tag{62}$$

While the result in Equation 62 appears to apply in the general case of the Cobb-Douglas, it must be remembered that Equation 62 is only true when $\alpha + \beta = 1$.

Constant Elasticity of Substitution
Production Functions

Suppose a product, or a group of products, is produced under conditions specified by the following production function:

$$Q = aL^{\gamma_1}\ K^{\gamma_2}\ M^{\gamma_3} \tag{63}$$

where K stands for the quantity of capital, L for the quantity of labor and M is the amount of raw material used (13). If the quantity of raw material is proportional to Q, and writing $\mu = M/Q$, then Equation 64 can be rewritten as:

$$Q = aL^{\gamma_1}\ K^{\gamma_2}\ (\mu Q)^{\gamma_3} \tag{64a}$$

$$Q^{1-\gamma_3} = a\mu^{\gamma_3}\ L^{\gamma_1}\ K^{\gamma_2} \tag{64b}$$

$$Q = (a\mu^{\gamma_3})^{\frac{1}{1-\gamma_3}}\ L^{\frac{\gamma_1}{1-\gamma_3}}\ K^{\frac{\gamma_2}{1-\gamma_3}} \tag{64c}$$

or

$$Q = a^{*} L^{\gamma_1 *} K^{\gamma_2 *} \qquad (64d)$$

where $\gamma_1 + \gamma_2 + \gamma_3 = 1$

and hence

$$\frac{\gamma_1}{1-\gamma_3} + \frac{\gamma_2}{1-\gamma_3} + \frac{\gamma_3}{1-\gamma_3} = \frac{1}{1-\gamma_3} \qquad (65)$$

Consequently, if Equation 63 is homogeneous of degree one then so is Equation 64d.

If W is the money wage per unit of labor and R the money rate of return on capital, then the distribution of the total product can be shown by

$$\$ \text{ Value added} = WL + RK \qquad (66)$$

If the values in Equation 66 are deflated by the current product price, P, and defining

$$V = \frac{\text{Value added}}{P}, \quad \frac{W}{P} = w \text{ and } R/P = r$$

Equation 66 can be rewritten, assuming pure competition, as

$$V = wL + rK = \frac{\partial Q}{\partial L} L + \frac{\partial Q}{\partial K} K \qquad (67)$$

If Equation 63 is differentiated with respect to L and K and the values substituted into Equation 67 the following is obtained:

$$V = \gamma_1 (aL^{\gamma_1 - 1} K^{\gamma_2} M^{\gamma_3}) L + \gamma_2 (aL^{\gamma_1} K^{\gamma_2 - 1} M^{\gamma_3}) K \qquad (68a)$$

or

$$V = (\gamma_1 + \gamma_2) Q \qquad (68b)$$

which can also be written as

$$V = (\gamma_1 + \gamma_2)\ a^* L^{\gamma_1} K^{\gamma_2} \qquad (68c)$$

by substituting the value of Q from Equation 64d into Equation 68b. However, since $\gamma_1 + \gamma_2 = 1$ it is clear that Equation 68c can also be written as:

$$V = AL^{\alpha} K^{1-\alpha} \qquad (69)$$

which is the Cobb-Douglas function with constant returns to scale.

The production function can be estimated for a product, or industry, if observations are available for V, K and L. Even though data are seldom available for all the variables, much information can be obtained from observations concerning only L, V and w.

If there is perfect competition among purchasers of labor, the pursuit of maximum profits will result in the workers being paid their marginal product or

$$w = \frac{\partial V}{\partial L}\ . \qquad (70)$$

If the production function is the Cobb-Douglas case, with constant returns to scale, the share of labor will be equal to the exponent of labor and

$$\alpha = w \frac{L}{V} \tag{71a}$$

or

$$\frac{L}{V} = \alpha w^{-1} \tag{71b}$$

Equation 71b written in logarithmic form becomes:

$$\log \left(\frac{L}{V}\right) = \log \alpha - \log w \tag{71c}$$

If this logarithmic function is fitted to observations on V, L and w, the hypothesis that the exponent of w in Equation 71b equals one can be tested. In statistical form Equation 71c, after inverting, becomes

$$\log \left(\frac{V}{L}\right)_i = - \left[\log a_i + b_i \log w_i + E_i\right] \tag{72}$$

If b turns out to be significantly different from one in Equation 72 the hypothesis that the Cobb-Douglas function described the production process would be rejected.

It should be emphasized that in the case of any linear homogeneous production function Equation 71a will always hold true. This is so because regardless of the value of α, the share of value added going to the other factor will always be $1 - \alpha$. Hence, it is clear that if b turns out not to differ statistically from zero, the fixed proportions production process would be accepted as an adequate description of the production process. In this case Equation 71b would imply that the ratio of labor to value added is always a constant regardless of the price of labor and that substitution is ruled out due to the technical nature of the production process.

The derivative of log (L/V) with respect to log w is b. Hence, b is the elasticity of labor input per unit of value added with respect to the wage rate w. It turns out on close examination that in the case of constant returns to scale, b is the same as the elasticity of substitution between labor and capital, σ, given in Equation 51. This fact can be demonstrated in the following manner. Suppose the production function to be homogeneous of degree one and represent it by

$$V = F(K,L). \tag{73}$$

then

$$\frac{V}{L} = F(\frac{K}{L},1) \tag{74}$$

but by rewriting

$$\frac{V}{L} = y \tag{75}$$

and

$$\frac{K}{L} = x \tag{76}$$

Equation 74 can be expressed as

$$V = Lf(\frac{K}{L}) = Lf(x) \tag{77}$$

When Equation 76 is differentiated partially with respect to L and K the following results are obtained:

$$\frac{\partial V}{\partial L} = f(x) + L \frac{\partial f}{\partial x} \frac{\partial x}{\partial L} \tag{78a}$$

or

$$\frac{\partial V}{\partial L} = f(x) + L \frac{\partial f}{\partial x} \frac{\partial (K/L)}{\partial L} \tag{78b}$$

but this is the same as

$$\frac{\partial V}{\partial L} = f(x) + Lf' \left(\frac{-K}{L^2}\right) = f - xf' \tag{78c}$$

Also,

$$\frac{\partial V}{\partial K} = L \frac{\partial f}{\partial x} \frac{\partial x}{\partial K} \tag{79a}$$

or

$$\frac{\partial V}{\partial K} = L \frac{\partial f}{\partial x} \frac{1}{L} = \frac{\partial f}{\partial x} = f' \tag{79b}$$

If pure competition exists among the buyers of labor the wage rate is equal to the marginal product of labor, hence,

$$w = f - xf' \tag{80}$$

and consequently

$$r = f' \tag{81}$$

Equation 78c specifies the relationship between w and y or V/L and taking the total derivative of Equation 80 with respect to w results in

$$\frac{dw}{dw} = \frac{df}{dw} - \frac{xdf'}{dw} - \frac{f'dx}{dw} \tag{82a}$$

$$1 = \frac{\partial f}{\partial x}\frac{dx}{dy}\frac{dy}{dw} - \frac{x\,\partial f'}{\partial x}\frac{dx}{dy}\frac{dy}{dw} - f'\frac{dx}{dy}\frac{dy}{dw} \tag{82b}$$

$$1 = f'\frac{dx}{dy}\frac{dy}{dw} - xf''\frac{dx}{dy}\frac{dy}{dw} - f'\frac{dx}{dy}\frac{dy}{dw} \tag{82c}$$

$$1 = -xf''\frac{dx}{dy}\frac{dy}{dw} \tag{82d}$$

$$-\frac{dw}{dy} = xf''\frac{dx}{dy} \tag{82e}$$

but

$$\frac{dx}{dy} = \frac{1}{f'} \tag{82f}$$

hence

$$-\frac{dy}{dw} = \frac{f'}{xf''} \tag{82g}$$

It is readily apparent, making use of the results in Equation 82g, that the elasticity of y or V/L with respect to w is given by

$$\frac{d \log y}{d \log w} = \frac{w}{y}\frac{dy}{dw} = \frac{f - xf'}{y}\left(- \frac{f'}{xf''}\right) \tag{83a}$$

$$\frac{d \log y}{d \log w} = - \frac{f'(f - xf')}{y\, xf''} \tag{83b}$$

or changing notation

$$\frac{d \log y}{d \log w} = - \frac{V_K\, V_L}{y\, x\, V_{KK}} \tag{83c}$$

or rewriting

$$\frac{d \log y}{d \log w} = - \frac{V_K\, V_L}{y\, V_{KL}} \tag{83d}$$

where

$$V_K = \partial V/\partial K \tag{84}$$

$$V_L = \partial V/\partial L \tag{85}$$

$$V_{KK} = \partial^2 V/\partial K^2 \tag{86}$$

but since

$$x = K/L \tag{87}$$

$$y = f(x) \tag{88}$$

$$f' = V_K \tag{89}$$

$$V_{KL} = f''\,\partial X/\partial L = f''(-K/L^2) \tag{90}$$

and hence

$$f'' = V_{KL}/(-K/L^2). \tag{91}$$

When these results are substituted into Equation 83a the following is obtained:

$$\frac{d \log y}{d \log w} = \frac{-V_K V_L}{y \frac{K}{L} \frac{V_{KL}}{\frac{-K}{L^2}}} = \frac{V_K V_L}{y V_{KL}} \tag{92}$$

and this is precisely the same result obtained in Equation 61, except for the change in notation, for the elasticity of substitution between labor and capital for a linear and homogeneous production function.

It is now readily apparent that estimates of σ, the elasticity of substitution, can be obtained by writing V/L or L/V as a function of w. Hence, estimates of σ can be obtained from empirical evidence and the use of Equation 72 rather than relying on the form of the hypothesized production function.

A linear and homogeneous production function, which contains the Cobb-Douglas and the constant proportions production functions as special cases and has a constant elasticity of substitution over all values of capital, can be written in one of its forms as:

$$V = \left[AK^{-\beta} + \alpha L^{-\beta}\right]^{-\frac{1}{\beta}} \tag{93}$$

where V, K and L have the same meaning as previously defined and A, α and β are the parameters of the production function. Minhas (13) refers to this function as the homohypallagic function.

The values of α and β are estimated by the results of Equation 72 and accordingly:

$$\log \alpha = \frac{-1}{b} \log a \tag{94}$$

$$\beta = \frac{-1}{b} - 1 \tag{95}$$

A can only be estimated if data on capital stock or rates of return are available and it is estimated by

$$A = [C]^{-\beta} \tag{96}$$

where

$$C = (V/K) \Big/ \left[1 - \frac{wL}{V}\right]^{\frac{1}{\beta}} \tag{97}$$

The elasticity of substitution is given by $\sigma = b$ from Equation 72 or by

$$\sigma = \frac{1}{1 + \beta} \tag{98}$$

from Equation 96.

Alternative forms of the CES function

An alternative form of the CES function is given by Arrow, Chenery, Minhas and Solow (14) as

$$V = \gamma\left[\delta K^{-\beta} + (1 - \delta)\, L^{-\beta}\right]^{-\frac{1}{\beta}} \tag{99}$$

where γ is termed the efficiency parameter, β is the substitution parameter and δ is the distribution parameter. The returns to a factor, given the value of β, will be determined by the distribution parameter. If neutral technological efficiency exists, the efficiency parameter is equal to unity.

β can range in value from -1 to ∞. If $\beta = -1$ the production process can be described by straight line isoquants. If b = 0 then $\beta = \infty$ and the production function is that of the fixed proportions variety and substitution is not possible. If b = -1 then $\beta = 0$ and the production process can be represented by the Cobb-Douglas function.

The equivalence, when $\beta = 0$, of Equation 99 and the Cobb-Douglas function can be demonstrated by rewriting Equation 99 in logarithmic form.

$$\log V = \frac{\beta \log \gamma - \log\left[\delta K^{-\beta} + (1 - \delta) L^{-\beta}\right]}{\beta} \tag{100}$$

Then making use of L'Hospital's Rule by taking the derivative of both the numerator and denominator of the right hand side with respect to β

$$\lim_{\beta \to 0} \log V = \lim_{\beta \to 0} \frac{\log \gamma - \dfrac{[-\delta K^{-\beta} \log K] + [-(1-\delta)L^{-\beta} \log L]}{\delta K^{-\beta} + (1-\delta)L^{-\beta}}}{1} \tag{101a}$$

$$\lim_{\beta \to 0} \log V = \lim_{\beta \to 0} \log \gamma + \frac{\delta K^{-\beta} \log K + (1-\delta)L^{-\beta} \log L}{\delta K^{-\beta} + (1-\delta)L^{-\beta}} \tag{101b}$$

$$\lim_{\beta \to 0} \log V = \lim_{\beta \to 0} \log \gamma + \frac{\delta \log K + (1-\delta) \log L}{\delta + (1-\delta)} \tag{101c}$$

$$\log V = \log \gamma + \delta \log K + (1-\delta) \log L \tag{101d}$$

$$\log V = \log (\gamma K^{\delta} L^{1-\delta}) \tag{101e}$$

$$V = \gamma K^{\delta} L^{1-\delta} \tag{101f}$$

and it is proved that the CES function reduces to the Cobb-Douglas function when $\beta = 0$.

The partial derivatives of Equation 99, when $\gamma = 1$, with respect to L and K results in

$$\frac{\partial V}{\partial L} = -\frac{1}{\beta}\left[\delta K^{-\beta}+(1-\delta)L^{-\beta}\right]^{-\frac{1}{\beta}-1}\left[-\beta(1-\delta)L^{(-\beta+1)}\right]=(1-\delta)\left[\frac{V}{L}\right]^{\beta+1} \tag{102}$$

$$\frac{\partial V}{\partial K} = -\frac{1}{\beta}\left[\delta K^{-\beta}+(1-\delta)L^{-\beta}\right]^{-\frac{1}{\beta}-1}\left[-\beta\,\delta\,K^{-(\beta+1)}\right] =\delta\left[\frac{V}{K}\right]^{\beta+1} \tag{103}$$

and

$$\frac{\partial^2 V}{\partial K\,\partial L} = \frac{1}{V}\left[(\beta + 1)\,\delta\,(\frac{V}{K})^{\beta+1}(1-\delta)\;(\frac{V}{L})^{\beta+1}\right] \tag{104}$$

The substitution of the above results into the formula for the elasticity of substitution given in Equation 61 results in

$$\sigma = \frac{1}{\beta + 1} \tag{105}$$

It is clear from Equations 102 and 103, given the value of β, that δ in fact does determine the functional distribution of income.

<u>Elasticity of substitution and relative factor shares</u>

Hicks (8) states that the relative share of a factor will increase if its elasticity of substitution exceeds unity and the use of the factor expands relative to the other factors. This relationship can be easily demonstrated for the CES function by the use of the formula for the elasticity of substitution. This is demonstrated as follows:

$$\sigma = V_K V_L \ / \ V V_{KL} \tag{106a}$$

and hence

$$(\sigma V_{KL} K) \ / \ V_L = (K V_K) \ / \ V \tag{106b}$$

denotes the relative share of capital, where V_K and V_L are the marginal products of capital and labor respectively and V is value added. The substitution of the partial derivatives from the CES function into the left hand member of Equation 106b, after simplifying and reversing terms, results in

$$(K V_K) \ / \ V = \frac{\delta \, (V/K)^{\frac{1}{\sigma}}}{(V/K)} \tag{107}$$

It is now clear that if σ equals one the relative share of capital is constant and equal to the parameter δ. If σ exceeds one, then the right hand side of Equation 107 increases as the ratio of value added to capital declines. This occurs when capital inputs are increased relative to labor and productivity of capital declines. Conversely, the relative share of capital decreases as its use increases relative to labor if the elasticity of substitution is less than one.

Hicks associated a declining marginal product with a declining elasticity of substitution but it is evident that he did not consider the Cobb-Douglas function. Hamberg (15) provides the example of

$$V = aK + bL + K^{\alpha} L^{1-\alpha} \tag{108}$$

where the elasticity of substitution increases as the marginal product of a factor declines. Hicks' reasoning that the elasticity of substitution must fall as the capital-labor ratio increased was that the most profitable substitutions would be made first and it would become increasingly difficult to make additional substitutions.

<u>Elasticity of substitution and technological advance</u>

If the elasticity of substitution exceeds unity and remains constant, the relative share of capital increases but the return per unit, or the marginal product, of capital will decline as the capital-labor ratio increases. Also, the elasticity of substitution could be less than one and constant as the capital-labor ratio increased, but the relative share of capital would decline while the marginal product of capital would also decline. In view of this information it is clear that technological advance must offset the declining marginal productivity of capital if investment is to be maintained in an industry irregardless of what is happening to the elasticity of substitution. Hence, it is the behavior of the marginal productivity of capital and not the behavior of its relative share that is important in maintaining a high level of economic progress. This is made more clear by pointing out that even though the relative share of one factor may be increasing, the absolute share of the other factor or factors could also be increasing.

The elasticity of substitution concept becomes important when the question arises of where should research and development funds be allocated to improve efficiency and to encourage investment. The following examples will make this more explicit: 1. If the elasticity of substitution and the rate of return on capital are high relative to other industries, why isn't investment occurring to lower the return? 2. If the elasticity of substitution and the rate of return on capital are both low, perhaps research should be directed to discovering new production processes to increase the substitution possibilities as well as the marginal productivity of capital. From these two examples it is clear that technological advance can induce investment and progress through permitting increased substitution and productivity of factors.

Under competitive conditions a high rate of return on capital would be expected to exist in an industry which sustained a level of technological advance greater than in other industries. That is, technological advance would occur at a rate fast enough to keep the flow of capital into the industry from depressing the rate of return on invested capital. Conversely, an industry with a low rate of technological advance might find the rate of return chronically depressed. However, since any measure of technological advance is necessarily restricted to that which is observed, an industry with a high interdependence among firms might display high profit rates and low technological advance.

A hypothesis which is suggested by the above reasoning is that industries with low rates of return have low rates of technological advance.

The effects of the price elasticities of the various goods has been largely ignored in the reasoning of the last two paragraphs. With other factors being equal, the higher the price elasticity of demand for a particular good or manufacturing service, the greater the inducement, or less the deterrent, for firms to increase the output of that particular good or service.

The problem of observed technological advance being so great as to increase output to the extent that prices fall and drag profits to low levels is also ignored. This can be justified on the basis that certain types of advance permit more efficient utilization of existing equipment and perhaps increase or at least hold profits constant as prices for finished goods or processing services fall. For example, computers permit more efficient inventory control and order filling. Labor might be replaced but less capital is also tied up in inventories and warehouse space.

Estimating Technical Change

Solow (7) has developed a method of estimating technological change in the aggregate production function but he suggests the function is actually more applicable to industry analysis. His method permits the estimation of the productivity change of labor when account is taken of the change in the quantity of capital available per worker. The estimation procedure which follows is that developed by Solow.

The production function in the Solow model is written as

$$Q = F(K,L;t) \tag{109}$$

where Q, K and L represent output, capital and labor respectively and t represents any kind of shift or change in the function.

If technological change is neutral, the variable t becomes a multiplicative factor which measures the cumulative effects of shifts over time. In this particular case the production function can be written as

$$Q = A(t)f(K,L) \tag{110}$$

When Equation 110 is differentiated totally with respect to t, which can be thought of as representing time as well as shift, and making use of the dot notation for a time derivative the following is obtained:

$$\dot{Q} = A \frac{df}{dt} + f \frac{dA}{dt} \tag{111}$$

but

$$A \frac{df}{dt} = A \frac{\partial f}{\partial K} \frac{dK}{dt} + A \frac{\partial f}{\partial L} \frac{dL}{dt} \tag{112}$$

hence

$$\dot{Q} = A \frac{\partial f}{\partial K} \frac{dK}{dt} + A \frac{\partial f}{\partial L} \frac{dL}{dt} + f \frac{dA}{dt} \tag{113}$$

If $\dot{K} = dK/dt$; $\dot{L} = dL/dt$ and $\dot{A} = dA/dt$ then Equation 113 becomes

$$\dot{Q} = A \frac{\partial f}{\partial K} \dot{K} + A \frac{\partial f}{\partial L} \dot{L} + f \dot{A} \tag{114}$$

and if Equation 114 is divided by Q with the three terms on the right hand side being divided and multiplied by K, L, and A respectively the following results:

$$\frac{\dot{Q}}{Q} = A \frac{\partial f}{\partial K} \frac{K}{Q} \frac{\dot{K}}{K} + A \frac{\partial f}{\partial L} \frac{L}{Q} \frac{\dot{L}}{L} + \frac{\dot{A}}{A} \tag{115}$$

and by defining $w_K = \frac{\partial Q}{\partial K} \frac{K}{Q}$ and $w_L = \frac{\partial Q}{\partial L} \frac{L}{Q}$ as the relative shares of capital and labor and noting that

$$\frac{\partial Q}{\partial K} = A \frac{\partial f}{\partial K} \tag{116}$$

hence on substituting

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + w_K \frac{\dot{K}}{K} + w_L \frac{\dot{L}}{L} \tag{117}$$

but

$$w_L = 1 - w_K \tag{118}$$

hence

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + w_K \frac{\dot{K}}{K} + \frac{\dot{L}}{L} - w_K \frac{\dot{L}}{L} \tag{119}$$

Let

$$q = \frac{Q}{L} \tag{120}$$

and

$$k = \frac{K}{L} \tag{121}$$

then taking the derivative of Equation 120 with respect to t produces

$$\dot{q} = \frac{L\dot{Q} - Q\dot{L}}{L^2} \tag{122}$$

and dividing both sides by q

$$\frac{\dot{q}}{q} = \left[\frac{L\dot{Q} - Q\dot{L}}{L^2}\right]\frac{L}{Q} = \frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} \tag{123}$$

When the result of Equation 123 is substituted into Equation 119

$$\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + w_K \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \tag{124}$$

hence

$$\frac{\dot{q}}{q} = \frac{\dot{A}}{A} + w_K \frac{\dot{k}}{k} \tag{125}$$

where $\dot{k}/k$ is derived in the same manner as $\dot{q}/q$.

So far technical change has been assumed to be neutral but if Equation 109 is considered instead of Equation 110 the result is very similar to the result shown in Equation 125. Rewriting Equation 109 for convenience

$$Q = F(K,L;\ t) \tag{126}$$

and differentiating totally with respect to time results in

$$\dot{Q} = \frac{\partial Q}{\partial K}\frac{dK}{dt} + \frac{\partial Q}{\partial L}\frac{dL}{dt} + \frac{\partial F}{\partial t} \tag{127}$$

and by making use of the same definitions and procedures as used from Equation 111 through Equation 125 results in

$$\frac{\dot{Q}}{Q} = \frac{\partial Q}{\partial K}\frac{K}{Q}\frac{\dot{K}}{K} + \frac{\partial Q}{\partial L}\frac{L}{Q}\frac{\dot{L}}{L} + \frac{\partial F}{\partial t}\frac{1}{F} \tag{128}$$

or simplifying

$$\frac{\dot{Q}}{Q} = \frac{\partial F}{\partial t}\frac{1}{F} + w_K\frac{\dot{K}}{K} + w_L\frac{\dot{L}}{L} \tag{129}$$

and hence

$$\frac{\dot{q}}{q} = \frac{\partial F}{\partial t}\frac{1}{F} + w_K\frac{\dot{k}}{k} \tag{130}$$

If $\dot{F}/F$ is independent of K and L, or with constant returns to scale K/L, then Equation 109 has the form of

Equation 110 and shifts in the production function are neutral. The demonstration of this involves the integration of a partial differential equation.

If $\dot{F}/F$ is equal to a constant, a, in time then $A(t) = e^{at}$ or in discrete form $A(t) = (1 + a)^t$.

When the shifts are neutral and constant returns to scale exist the production function can be represented by a graph of q plotted against k. This is so because if the unit-output iso-quant is known the whole map is known. A difficulty arises, however, because the production function is shifting over time so that observed points in the (q,k) plane are compounded out of movements along the curve as well as shifts of the curve. In Figure 3 each ordinate on the curve labeled t = 2 is a result of the curve labeled t = 1 being multiplied by a constant to bring about a neutral upward shift in the production function at time period two. The problem is to estimate this shift factor from observed points on the two curves.

If the production function was fitted through the points P_1 and P_2, no allowance would be given for the fact that the capital-labor ratio had changed between the two time periods. However, if a shift factor can be estimated for each time period the observed points can be corrected for technical change and the production function can be estimated.

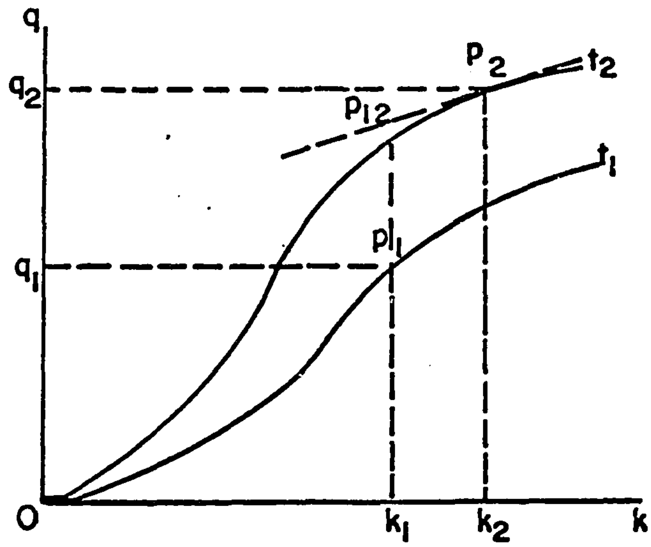

Figure 3. Production functions illustrating technological change

For small changes a line tangent to point P_2 can be constructed and $\Delta A/A$ can be estimated by the ratio of $P_{12}P_1/q_1$ where the distance P_1P_{12} represents the upward shift in the function. An interesting point is the fact that

$$k_1 P_{12} = q_2 - \partial q/\partial k \Delta k \tag{131}$$

and consequently

$$P_{12} P_1 = q_2 - q_1 - \partial q/\partial k \Delta k = \Delta q - \partial q/\partial k \Delta k \tag{132}$$

and

$$\Delta A/A = P_{12}P_1/q_1 = \Delta q/q - \partial q/\partial k(k/q)\Delta k/k = \Delta q/q - w_K \Delta k/k \tag{133}$$

and Equation 125 can be rewritten as

$$\frac{\dot{A}}{A} = \frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k} \tag{134}$$

Hence, Equation 133 represents the discrete approximation to Equation 134.

If arbitrarily A(t) = 1 and

$$A(t + 1) = A(t)\ (1 + \Delta A/A) \tag{135a}$$

then

$$A(t + n) = A(t)\ (1 + n\ \Delta A/A) \tag{135b}$$

and Equations 134, 135a and 135b can be used to estimate the technological change parameter.

Solow used Gross National Product exclusive of government and agricultural output. His capital series excluded capital from government, agriculture, and consumer durables. The capital input was capital stock in use rather than an annual flow of capital services. As a point of reference, output per man-hour increased from about $.623 to $1.275 from 1909 to 1949 with the cumulative shift in the production function being 1.809 or about 80 percent. The Gross National Product, net of technical change is $1.275 ÷ 1.809 or about $.705. That is to say, if technical change had not occurred output per man-hour would only have been $.705 in 1949. Hence, $.705 - $.623 or about $.08 per man-hour must be due to increased use of capital per worker.

METHODS AND PROCEDURES

This study, like many others based on aggregate data, had to rely occasionally on sources of data which left something to be desired. As a consequence, some adjustments were made in the data in an effort to improve its usefulness. While the reason or need for making adjustments in data are usually fairly apparent, it is not always easy to carry out the mechanics of adjustment in a fashion that satisfactorily bridges the gap between that which is available and that which is desired. There are also adjustments which the researcher would desire to make in his data but the cost involved may outweigh the benefits to be derived. In other cases the researcher may be at a loss to know in which direction to make the adjustment, that is, should the data be adjusted upwards or downwards.

Census of Manufactures

Data concerning value added, payrolls, and employees were obtained from the Census of Manufactures (16) and in a few instances from the Survey of Manufactures (17). All of the census years between 1925 and 1958 were included in the analysis.

Value added

Value added is defined by the United States Department of Commerce as the value of shipments minus cost of materials, supplies, containers, fuel, purchased electric energy and contract work. It avoids the duplication in value of shipment statistics which would result from the use of output of one manufacturing plant as materials by another plant. The value added statistics generally are not adjusted for changes in inventories. However, inventory changes have been reported at the national level for some census years but not on a state or regional basis.

Employees

The term all employees as used by the Department of Commerce includes all full-time and part-time employees on the payrolls of operating manufacturing establishments who worked or received pay for any part of the pay period ending nearest the 15th of the months specified on the report forms. These forms are currently filled out four times each census year and prior to 1954 they were collected each month of the census year. Officers of corporations are included as employees but proprietors and partners of unincorporated businesses are not. Employees at separate administrative offices and auxiliary units are also excluded from the total.

Payrolls

Payrolls included the gross earnings paid to all employees on the payroll of the manufacturing establishments. It does not include the employer's contributions to pension and other fringe benefit plans. Also, the amounts paid to administrative employees at separate central administrative offices and auxiliary units, as well as proprietors and partners salaries, are omitted.

Internal Revenue Service

The original source of all capital data used was the Source Book of Statistics of Income (18) which is a compilation of worksheets kept by the Internal Revenue Service. The Source Book contains income statement and balance sheet items distributed by total asset size of corporations. The data in the Source Book are compiled by stratified sampling from corporation income tax returns filed at the 62 Internal Revenue Districts. The returns are stratified by the size of total assets with all of the corporations with large total assets or large net income or deficit being included in the sample.

Stigler (19) has compiled a capital series on the three-digit industries in the Food and Kindred Products Industry starting in 1938 and ending in 1957. He adjusted the corporation data to include estimates of the assets of the non-corporate segments of the industries. His series was then extended, for the purposes of this study, to include data from the 1961 corporation income tax returns which are the latest returns available from the *Source Book*. The capital figures are all in book values.

Capital measurement issues

The method of including capital in production function and productivity estimate studies has provoked some arguments and comments among workers in the field. For example, in what units should capital be measured? Should capital inputs be measured in so many pieces of a well defined component of equipment such as an electric motor, or so much horsepower, or so many dollars worth of motors in the production unit? If the capital input is to be measured in dollars, is the gross value to be used, depreciated value, or flow of capital services? The appropriateness of each will depend upon the efficiency of the particular machine with advancing age and the rate of obsolescence which reflects technological advance. As a matter of convenience and feasibility, total value of capital in use has been the most common measure of capital for productivity studies.

Ideally, one would like to deflate book values of capital to reflect the changing price level of machinery and equipment, buildings, inventories, other working capital and land. The implicit price index given in the Economic Report of the President (20) indicates that the price of manufacturing machinery and equipment has risen fairly steadily over the last several decades. However, deflation of the book values of assets is not a simple matter of dividing the book values by the price index. The reason being that the book values represent a composite of book values of machinery and equipment acquired over a period of years at different price levels. Consequently, a composite index must be used to deflate that proportion of total book value contributed by purchases in each preceding year. The hypothetical example shown in Table 1 for machinery and equipment will be useful to illustrate the principles involved.

In this particular illustration it is assumed that the machinery and equipment has a useful life of five years and that expenditures in varying degrees are made for its replacement each year. The purchase price of the 20 percent purchased in 1961 that is in use today is not deflated by the index of today's machinery and equipment prices but by the price index of 1961. The index to deflate the 1965 book value is obtained by dividing 32.36 by 38.00 and the result is a composite price index in 1965 of about .85. The same procedure would apply to derive the composite index for deflating the 1964 book values except the series would start in 1960 and end in 1964.

Table 1. Illustrative calculation of price index of book value of machinery and equipment

Year	Percent in use 1965 (1)	Purchases of machinery and equipment (2)	Weighted by percent in use 1965 (1) x (2) (3)	Price index 1965 = 100 (4)	Purchases of machinery and equipment in current prices weighted by percent in use 1965 (3) x (4) (5)
1961	20	10	2.00	60	1.20
1962	40	11	4.40	70	3.08
1963	60	12	7.20	80	5.76
1964	80	13	10.40	80	8.32
1965	100	14	14.00	100	14.00
			38.00		32.36

The Department of Commerce has compiled data on expenditures for machinery and equipment for all manufacturing industries as a group. However, information at the three-digit level is insufficient to construct a composite index, as expenditures on machinery and equipment for a period of 30 years would be required to construct a capital series with 15 observations. This is so because the average length of life of machinery and equipment, as given by the Internal Revenue Service, is 15 years. Since it will be impossible to deflate the book values of capital for the three-digit industries, the next best alternative is to determine under what circumstances use of the book values is meaningful.

If all prices remained unchanged and no improvements were ever made in the machinery and equipment, or the effectiveness with which it is used, and the procedures used by accountants reflected exactly the decline in value due to use, then the book value would reflect the replacement or market value of the machinery and equipment. If everything mentioned above remained constant except the effectiveness with which the machinery is used, the added output from the same amount of inputs as previously used can be credited to technological advance. If, in addition, the efficiency of the machine is enhanced with everything else remaining constant, the increase in output due to this source can also be attributed to technological advance. If changes in book values of capital per worker can be accounted for over time, then changes in output can be attributed to either increased capital per worker or technological advance. Also, if comparisons are being made between industries that use similar accounting practices due to trade association programs, Internal Revenue Service regulations, or similarity of processes and techniques, then comparisons on productivity changes due to technological change are meaningful.

The changing price level of output does not pose a problem in estimates of technological change as the change in output due to technological change and change in output due to increased capital per worker are both effected in the same proportion.

Stigler (19) prepared a capital series for Food and Kindred Products in constant 1947 prices by using the annual investment in all manufacturing industries for his annual weights instead of investment in the Food and Kindred Products Industry. He warned that this was a wide gap to bridge and stated that it would be meaningless to apply the same procedure to any three-digit industry. The technological change parameter was estimated using his series in constant 1947 dollars in order to compare the results with the parameters computed from book values.

The capital series from the Internal Revenue Service and the data on labor inputs and value added from the Census and Survey of Manufactures were used to estimate the technological change from 1939 to 1961. Even though these sources of data are not strictly comparable, they are the best estimates available and will have to carry the burden until better sources are devised.

The percentage rates of return on total assets were obtained from Stigler's valuable work and the Source Book of Statistics of Income. The return on capital is composed of the sum of all interest payments to lenders, dividends on all types of stock, and undistributed profits. This figure is net of all corporate income tax, other business taxes, and does not include dividends received from other corporations. The measure of assets was total assets less investments in other corporations. The percentage return is thus different from that which would have been obtained if stockholders' equity had been used for the measure of assets.

Units of Observation

The choice of the unit to be observed or studied sometimes poses one of the major problems to the researcher. For example, in a cost study or production function study, should the unit of inquiry be a stage of production, a process, a plant, the firm, the industry, or the whole nation? Ideally the researcher should have a choice of the unit to be studied that would be determined by the particular objectives of the research. In some cases the researcher may be forced to observe aggregates and make inferences from the aggregate data to the individual units. In other instances the researcher may be forced to view individual units and infer from their behavior or characteristics to the aggregates. If the relationship between the aggregates and the individual units is one of simply adding up the relevant quantities with all parameters of the aggregates being equal to, or a multiple of, the parameters of the individual units, no problems would arise in choosing the appropriate unit of observation. However, it is not always clear that the behavior, or the characteristics, of the group will resemble that of the individual units. For example, in purely competitive industries the demand elasticity for output facing the firm and industry are different while the supply elasticity for inputs can be the same in many instances.

If the units observed, say at the plant level of an industry, represent a wide variety of current techniques the result will be an average function and may or may not describe any actual plant in operation. The observing of a cross-section in this fashion would not describe or specify the relationship between inputs and outputs that would be of interest to an investor seeking investment opportunities. However, the policy maker seeking improved productivity of resources should be very much interested in the average performance of the industry. Hence, the study of the aggregate or industry is oftentimes the relevant unit to be observed from the policy makers viewpoint.

The relationships observed for the industry should describe the locus of the firm equilibrium conditions. Hence, under certain conditions the iso-quants or the factor-product relationships can be estimated for a product. Equation 72 was used to estimate the locus of the factor-product equilibrium points and the results were used to make inferences about the iso-quants. It was proved in Equations

73 to 92 that the regression coefficient of Equation 72 was equal to the elasticity of substitution of labor for capital. The conditions that must be met are as follows: 1. a linear homogeneous production function; 2. firms were profit maximizers; 3. firms were price takers in factor markets; 4. workers were paid according to their marginal value productivity; 5. that all firms have the same technological knowledge; 6. prices of output and raw material inputs do not vary systematically with wage rates; 7. the firms are in equilibrium with respect to the optimum combination of inputs.

Efforts were made to estimate the elasticity using data aggregated at either the nine census divisions or the four census regions for the census years 1958, 1954 and 1948 as well as at the state level for all census years between 1925 and 1958. The census data for 1958 are the latest data available.

The data aggregated at the census division or region level were adjusted for changes in inventory, the opportunity cost of wages and salaries for proprietors and partners, administrative personnel at separate locations, and fringe benefits. The Census of Manufactures gave totals for the United States for some of the adjustment factors and these were distributed over the regions by the ratio of value added in the region or division to the United States total.

Inventory changes were not given for all years and, consequently, it was not possible to make all the desired adjustments.

Even though the numbering scheme of the Census of Manufactures has changed over the period of years covered in this study, the same set of numbers was used throughout for purposes of simplification. Also, the names used by the Department of Commerce have changed from census to census. The reader may consult Appendix A for a complete matching of names and numbers to compare the name used by the Department of Commerce to the number attached to that industry for the purposes of this study. Since all of the four-digit data were used in cross-section analysis only, no great harm was done by arbitrarily using the same set of numbers throughout the time period involved.

It should be mentioned that even though the census data do not meet the requirements of least squares analysis in order to test hypotheses concerning r^2's and b values, the data were treated as being a sample meeting the requirements for the purposes of this study. This is the usual procedure in economic analysis when better methods are unavailable. Even though the data are called census data, they are composed of estimates made by the respondents to the questionnaires in many cases.

Tests of hypotheses

The standard hypotheses tested were that r^2 equaled zero; π, the population regression coefficient, equaled zero and one; π was greater than one; and that π was greater than zero and less than one. The test that π equaled zero was a one-tailed t-test as negative values of π were ruled out by the model, consequently, the test that r^2 equaled zero was also performed. The test that π equaled one was a two-tailed test as π can be greater than one. If π was different from zero and one, and the value of b was between zero and one, then π was said to be between zero and one. This particular procedure results in unequal probabilities being in each tail of the distribution but the probabilities involved should be clear from the context. Occasionally a b value did not differ statistically from either zero or one.

Four simple regression models were used to test the hypothesis that the rate of return on total assets was positively correlated with the rate of technological advance. The 1939 observations are omitted because of the length of the period between 1939 and 1948. These models hypothesize that the change in the rate of return between two different years, ΔZ, is a linear function of the change in A(t), the technological change parameter, either for the same period or of the preceding period.

The hypothesis that the relative share of labor had remained constant over the time period of the study was tested by a simple linear regression equation which used time as the independent variable and the relative share in each census year as the dependent variable.

RESULTS

The detailed results of fitting Equation 72 to the Census of Manufactures data aggregated at the state level are shown in Tables 11 to 20 and are contained in Appendix B. These results are summarized in Table 2. The information presented in these tables is as follows: the industry number; the year; N, the number of observations; r^2, the percentage of the total variance explained by the regression equation; F, the level of significance of the F-test that r^2 equaled zero; log a, the constant term of the regression equation; b, the sample regression coefficient; the level of significance of the t-test that π, the population regression coefficient, equaled either zero or one, or was in the interval zero to one; and the ratio of the variables V/L, W/L, and W/V. In addition, the standard errors of the variables and estimators are shown in parentheses beneath the appropriate estimator or variable.

The results of fitting Equation 72 to the data aggregated at the census division and region level are summarized in Table 3 but are omitted from the appendix due to the lack of significant results.

The estimates of technological change, along with the rates of return on total assets, are given in Tables 21 to 28 and are contained in Appendix C. Table 4 summarizes the estimates of technological change while Tables 6 to 9 summarize the results of the tests of the relationship between profits and technological change.

Table 10 contains the results of the test of the hypothesis that the relative shares of labor were constant over the time period of the study.

Estimates Derived from State Data

As Table 2 indicates, a fairly consistent pattern of elasticities of substitution of labor for capital exists in the Food and Kindred Products Industries. Some strong tendencies and patterns appear to exist for the group as a whole as well as for some of the individual three- and four-digit industries.

Since the four-digit industries are more homogeneous than the three-digit industries, the main emphasis will be on the results obtained from four-digit data. The three-digit data are a summation of the four-digit data for each state, except where data were withheld at the four-digit level and sometimes three-digit data were withheld as well. Hence, the three-digit estimates would be an average of the four-digit results. The discussion of the results will be limited to the more interesting and most convincing results. However, the tables do contain all the information that was derived from the study.

Table 2. Summary of statistical results of state data

Industry number	Number of years	r^2's different from zero	π[a] different from zero	π different from one	π greater than one	π between zero and one
201	9	8	7	8	0	6
2011	10	7	7	4	0	1
2013	9	6	6	4	0	1
2015	8	6	6	1	0	0
202	5	5	5	2	0	2
2021	10	4	4	3	0	0
2022	10	3	3	2	0	0
2023	10	2	2	3	0	0
2024	10	10	10	0	0	0
2025	4	0	0	0	0	0
2026	2	2	2	0	0	0
203	4	4	4	0	0	0
2031	10	10	10	2	2	0
2032	4	1	1	0	0	0
2033	10	9	9	4	3	1
2035	4	2	2	0	0	0
2036	2	2	2	0	0	0
2037	4	2	2	1	0	1
204	10	10	10	1	0	1
2041	9	8	8	1	1	0
2042	9	8	8	3	0	1
2043	3	2	2	0	0	0
2044	9	3	3	2	2	0
2045	4	0	0	0	0	0
205	4	4	4	2	0	2
2051	10	10	10	7	0	7
2052	5	3	3	0	0	0
205XX	2	2	2	1	0	1
206	3	0	0	0	0	0
2062	6	0	0	1	0	0
2063	8	0	0	0	0	0

[a] π is the population regression coefficient.

Table 2. (Continued)

Industry number	Number of years	r^2's different from zero	π different from zero	π different from one	π greater than one	π between zero and one
207	4	3	3	1	0	0
2071	9	8	8	1	0	0
2072	8	0	0	0	0	0
2073	1	0	0	0	0	0
208	7	6	6	4	2	1
2081	6	5	4	4	0	4
2082	7	7	7	0	0	0
2083	7	0	0	1	0	0
2084	5	0	0	0	0	0
2085	5	1	1	0	0	0
2086	3	1	1	1	0	0
2087	9	4	4	1	0	0
209	5	4	4	0	0	0
2091	1	1	1	0	0	0
2092	4	1	1	1	0	0
2093	2	0	0	0	0	0
2094	2	0	0	1	0	0
2095	1	0	0	0	0	0
209XX	2	0	0	0	0	0
2096	7	2	2	2	0	1
2096X	5	2	2	0	0	0
2097	10	10	10	4	0	4
2098	8	7	7	1	1	0
2099A	1	0	0	0	0	0
2099	8	8	8	0	0	0
3000	3	2	2	0	0	0
3001	2	2	2	0	0	0
3002	2	0	0	0	0	0
3003	1	1	1	0	0	0

Industry 2011

Industry 2011 is currently called Meat Packing by the Census of Manufactures and it is the industry that purchases livestock and primarily sells animal carcasses but may also do some further processing. Seven of a possible ten b values differed statistically from zero, four b's differed from one, and one b value was between zero and one.

Industry 2013

This is the Prepared Meats Industry and it primarily buys carcasses from firms or plants in Industry 2011 for further processing but it may also do some slaughtering of its own. Six b's out of a possible nine differed statistically from zero, four differed from one, and one was in the interval zero to one.

Industry 2015

This industry is currently named Poultry Dressing, Wholesale. Six of a possible eight b values differed statistically from zero with none of the six different from one. The one b value which was different from one was not different from zero and it occurred in 1925.

All b values since 1935 differed from zero with none different from one. Hence, there is strong evidence to indicate that this industry can be described by the

Cobb-Douglas function.

Industry 2021

The results indicate that the elasticity of substitution of labor for capital in the Creamery Butter Industry has fluctuated considerably over the time period of the study. Four b values differed statistically from zero and three differed from one with none in the interval zero to one.

Industry 2022

This is the Natural Cheese Industry and here, too, the results indicated considerable stability in the elasticity of substitution of labor for capital. Only three b values of a possible ten differed from zero, two b's differed from one and none were in the interval zero to one.

Industry 2023

The results for the Condensed and Evaporated Milk Industry were also quite stable as only two out of ten b values were statistically different from zero. Apparently this industry has had fairly stable technological conditions with substitution possibilities very low or impossible.

Industry 2024

The model produced ten out of a possible ten b values significantly different from zero in the Ice Cream and Ices Industry. In addition, none of the ten b values were significantly different from one. Hence, the evidence strongly indicates that this industry has the Cobb-Douglas production function.

Industry 2026

This is the Fluid Milk Industry and observations are only available for two different years. However, both b values differed from zero with neither different from one. Hence, this industry seems to have the Cobb-Douglas production function.

Industry 203

Results for the Canning, Preserving and Freezing Industry indicate that the Cobb-Douglas function can adequately describe its production function. It was only possible to combine the four-digit industries into a meaningful aggregate for four different years. However, all four b values differed from zero and none differed from one.

Industry 2031

This is currently named the Canned Sea Foods Industry and it, too, can be described by the Cobb-Douglas function. Ten out of ten b values differed from zero and only two differed from one. The two b values different from one were greater than one and this is one of the few times in the study that a b value exceeded one. Only nine b values in the entire study exceeded one.

Industry 2032

The results for this industry, Canning and Preserving except Fish, followed about the same pattern as the results for Industry 2031. It had three of a possible four b values exceeding one and two of these values occurred in the same years as the b values greater than one in Industry 2032. These two years were 1954 and 1939 with the third year being 1925. Perhaps something occurred in the technology of canning and preserving that affected the elasticity of substitution of labor for capital in both of these industries at about the same time.

Industry 2036

This is the Fresh or Frozen Packaged Fish Industry. Observations are only available for two years but both b values differed significantly from zero with neither different from one. Hence, the evidence indicates that this industry can be described by the Cobb-Douglas function.

Industry 2041

This industry is entitled Flour and Meal and eight out of nine b values differed statistically from zero and only one b value, that for 1937, differed from one. The b value for 1947 did not differ from either zero or one. The b value for 1937 was greater than one. Grain Production was low in both years and the value added statistics might have been distorted in both years due to inventory changes.

Industry 2042

Eight out of nine b values differed significantly from zero with the b value not different from zero occurring in 1931 in the Prepared Animal Feeds Industry. Six of the nine b values were not statistically different from one. The b values for 1935 and 1937 were in the interval zero to one. The b value for 1931 was not significantly different from zero but was different from one. The evidence at hand indicates that this industry, as well as Industry 2041, could best be estimated by a Cobb-Douglas function part of the time and had elasticities between zero and one part of the time. This is particularly interesting as the technological processes involved are identical for these two industries in many of the production stages and very similar in other stages.

Industry 2043

This was the Cereal Breakfast Foods Industry and separate data have not been compiled for it since 1954. Only three observations were available for 1954 and no significant results occurred for that year. The b values were both statistically different from zero and neither differed from one. Hence, this processing industry can also be described by the Cobb-Douglas function for the years 1931 and 1933.

Industry 2071

This is currently entitled the Confectionery Products Industry and eight of a possible nine b values were statistically different from zero, with one b value being different from one but not different from zero. This latter case occurred in 1925 but since then the behavior of the industry has conformed to the Cobb-Douglas production function.

Industry 2081

The results for the Beverages Non-alcoholic Industry contained five out of six b values statistically different from zero, with four different from one and three in the interval zero to one.

Industry 2082

This is the Malt Liquors Industry and seven out of seven b values are statistically different from zero and not different from one. Hence, we would conclude that this industry can be adequately described by the Cobb-Douglas production function.

Industry 2091

Observations were only available for 1958 for the Cotton Seed Oil Meal Industry as it previously was not classified in Food and Kindred Products. However, the b value indicates a high probability that the capital-labor substitution relationship can be estimated by a Cobb-Douglas production function.

Industry 2097

The Manufactured Ice Industry had ten out of ten b values statistically different from zero with four lying in the interval zero to one with the rest not different from one.

Miscellaneous Industries

Some of the smaller industries had only limited information available but several of these, arbitrarily given the numbers 3000, 3001, 3002, and 3003, gave some indication of possessing an elasticity of substitution of labor for capital of one, and hence, manifesting a Cobb-Douglas production function.

Consistent Patterns

An examination of Table 2 will show that only nine b values were significantly greater than one. Of these nine, five appeared in two four-digit industries with similar processes. Also, four of these values were in the same years. This is powerful evidence that, except perhaps under severe disturbances, the elasticity of substitution of labor for capital lies consistently within the interval zero and one.

Occasionally, a b value had a negative sign but none of these are statistically different from zero. In general, these negative b values occurred in instances when very few observations were available.

An examination of the scatter diagrams of V/L against W/L reveals that in many cases a very large part of the variance is contributed by a relatively small number of observations. Some possible reasons for some of the observations diverging so widely from the others, besides mistakes in recording of the census data, could be: 1. extreme differences in the techniques of production between states because of the difference in construction and installation dates of plant and equipment; 2. the nature of the product may be radically different in some states from the rest of the states even though the Census of Manufactures classification scheme places them in the same industry group; 3. extreme variations in raw material availabilities due to weather conditions; 4. the failure of the Census of Manufactures data to take account of inventory changes between the beginning and ending of the census year.

The variation in raw materials prices will affect the observed values less than one would at first suppose. The reason is that the value added figure is net of raw materials and the value of the finished product should increase along with the price of raw materials. Hence, the data, to a certain extent, should be self-adjusting for variations in prices of raw materials and of finished products. If a firm, or the firms within a state, accumulated large quantities of raw materials and finished products in certain years relative to firms in other states, then the observed values of V/L for that state would be distorted downward. The value of V/L would be distorted upward if the firms within a state depleted inventories of raw materials and finished goods.

The scatter diagram for Industry 204, Grain-Mill Products, and Industry 2042, Prepared Animal Feeds, reveals that the states, Maryland and Delaware, have extremely high values of V/L relative to W/L for the Census years starting in 1948. This could be attributed to the fact that the development of the poultry industry in those states brought about the construction of highly automated feed mills.

Estimates Derived from Census Division and Region Data

Table 3 summarizes the results of the efforts to estimate the elasticity of substitution of labor for capital from data aggregated at the census division and region level for the years 1947, 1954 and 1958. In general, very poor statistical results were obtained from these efforts as indicated by Table 3. Possible reasons for these poor results might be attributed to the diverse conditions found within a division and region. Consequently, when the data were aggregated any relationship existing among the variables were apparently masked in the process of aggregating. A relationship which did emerge, however, was the lack of elasticity coefficients which exceeded one in value. Hence, additional evidence is obtained to demonstrate the consistency with which the elasticity coefficient lies between zero and one.

Estimates of Technological Change

Table 4 contains the estimates of technological change. The variables determining this parameter and the detailed computations, as well as the rates of return on total assets, are shown in Appendix D. The values for the technological change parameter range from a low of 1.575 for the Dairy Products Industry to a high of 3.307 for the Sugar Industry. The industries possessing the second and third highest rates

Table 3. Summary of statistical results of census division and region data

Industry number	Number of years	r^2's different from zero	π^a different from zero	π different from one	π greater than one	π between zero and one
201	3	2	2	2	0	1
2011	3	2	2	0	0	0
2013	3	0	1	2	0	0
2015	3	1	1	1	0	1
202	3	2	3	0	0	0
2021	3	0	0	0	0	0
2022	3	0	1	1	0	0
2023	3	0	0	0	0	0
2024	3	1	1	0	0	0
2025	3	0	0	0	0	0
2026	2	1	2	0	0	0
2027	1	0	1	0	0	0
203	3	1	1	0	0	0
2031	3	0	1	0	0	0
2032	3	1	2	0	0	0
2033	3	3	3	1	1	0
2034	3	0	0	0	0	0
2035	3	2	2	0	0	0
2036	2	1	1	0	0	0
2037	3	2	2	0	0	0
204	3	2	2	0	0	0
2041	3	3	3	0	0	0
2042	3	2	3	0	0	0
2043	3	0	0	0	0	0
2044	0	0	0	0	0	0
2045	3	1	3	0	0	0
205	3	2	3	0	0	0
2051	3	2	3	0	0	0
2052	3	2	2	1	1	0
206	2	0	0	0	0	0
2062	3	0	0	0	0	0
2063	1	1	1	1	1	0

[a]π is the population regression coefficient.

Table 3. (Continued)

Industry number	Number of years	r^2's different from zero	π different from zero	π different from one	π greater than one	π between zero and one
207	3	1	1	0	0	0
2071	3	1	1	0	0	0
2072	0	0	0	0	0	0
2073	0	0	0	0	0	0
208	3	2	2	0	0	0
2081	2	2	2	1	0	1
2082	3	2	2	0	0	0
2083	3	0	0	0	0	0
2084	3	1	1	1	0	1
2085	3	0	0	0	0	0
209	3	1	2	0	0	0
2091	2	1	1	0	0	0
2092	3	0	1	1	0	0
2093	2	0	0	0	0	0
2094	3	1	1	0	0	0
2095	3	0	0	0	0	0
2096	2	1	1	0	0	0
2097	3	3	3	1	1	0
2098	3	1	3	1	1	0
2099A	1	0	1	0	0	0
2099	3	2	3	2	2	0

Table 4. Technological change parameters

Industry number / Year	20[a] A(t)	20 A(t)	201 A(t)	202 A(t)	203 A(t)	204 A(t)	205 A(t)	206 A(t)	207 A(t)	208 A(t)	209 A(t)
1939	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1947	2.040	1.728	1.792	1.392	1.760	1.752	1.592	1.776	1.792	1.576	1.856
1949	2.150	1.724	2.011	1.484	1.658	1.647	1.703	1.755	1.577	1.516	1.581
1950	2.141	1.708	1.906	1.279	1.918	1.728	1.700	1.702	1.645	1.471	1.695
1951	2.267	1.730	2.036	1.303	1.887	1.785	1.804	1.876	1.584	1.330	1.792
1952	2.419	1.830									
1953	2.615	1.945	2.350	1.590	2.102	1.903	1.941	1.902	1.733	1.543	1.896
1954	2.738	2.025	2.263	1.482	2.072	1.800	1.968	2.045	1.813	1.443	2.002
1955	2.982	2.171	2.607	2.002	2.128	1.917	2.064	2.405	1.907	1.583	2.090
1956	3.220	2.288	2.774	1.930	2.420	1.877	2.178	2.821	1.974	1.684	2.207
1957	3.365	2.343	2.863	2.154	2.139	1.930	2.256	3.044	2.114	1.664	2.269
1958		2.294	2.771	1.499	2.500	2.131	2.364	2.761	2.148	1.639	
1959		2.347	2.829	1.547	2.555	2.108	2.456	2.946	2.174	1.724	
1960		2.467	2.877	1.593	2.793		2.485	3.364	2.174	1.765	
1961			2.871	1.575	2.949	2.159	2.435	3.307	2.235	1.677	

[a]The technological change parameter in this column was derived from a capital series in constant 1947 dollars.

Table 5. Proportion of output gains due to technological change and increased use of capital

Industry	A(t)	Q/L 1939 ($1000)	Q/L 1961 ($1000)	Q/L ($1000)	Change due to technological advance (percent)	Change due to increased capital (percent)
201	2.871	2.414	9.045	6.631	88.9	11.1
202	1.575	3.281	11.548	8.267	50.9	49.1
203	2.949	2.097	10.191	8.094	83.2	16.8
204	2.159	4.434	17.666	13.232	71.7	28.3
205	2.435	2.689	9.630	6.941	81.8	18.2
206	3.307	3.366	13.415	10.049	93.1	16.9
207	2.235	2.794	11.032	8.238	74.0	26.0
208	1.677	4.988	15.520	10.532	59.4	40.6

of change are the Canning, Preserving and Freezing Industry, and the Meat Products Industry.

Table 5 summarizes the change in output per man-year between 1939 and 1961. The total change is divided into a component due to the increased use of capital per man and a part due to technological change. It is especially interesting to note that the industry with the lowest rate of technological advance had over 50.9 percent of its output gain due to technological change. Hence, the role of technological advance in securing increased productivity is shown to be very important.

Table 6. Relationship between ΔA(t) and ΔZ in all industries

Year	N	r^2	a	b (s_b)	$\pi = 0$
1949	8	.0080	- .1637	.7466 (3.3849)	
1950	8	.5723	.1399	9.7046 (3.4253)	**
1951	8	.4156	-2.4089	8.5582 (4.1438)	
1953	8	.2955	-1.3306	6.7468 (4.2521)	
1954	8	.0364	- .1485	1.7004 (3.5696)	
1955	8	.0860	1.2325	-2.1773 (2.8983)	
1957	8	.3320	- .3992	2.6552 (1.5373)	
1958	8	.1864	.0614	1.3662 (1.1653)	
1959	8	.1347	.5408	-3.2638 (3.3767)	
1960	8	.6285	- .0173	-6.4669 (2.0298)	**
1961	8	.0124	.4067	-2.1134 (7.7043)	

Four simple regression models were used to test the hypothesized relationship between technological change and the rate of return on total assets.

The first model tests the hypothesis that the change in the rate of return between succeeding years, ΔZ, is a linear function of the change in the technological change parameter, ΔA(t) for the same time period. The results of this test are shown in Table 6. The only significant results obtained were for the years 1950 and 1960. However, the sign of the regression coefficient for the latter year was negative. In addition, two other regression coefficients were negative but nine out of a possible 12 had positive regression coefficients.

The second model tests the hypothesis that the year to year differences in the rate of profit within an industry is a linear function of the variation in the technological change parameter for the same time span. These results are presented in Table 7. The only significant result was for Industry 207. However, the sign of all except two regression coefficients was positive.

The third model tests the hypothesis that the variation in the rate of return in all industries is a linear function of

Table 7. Relationship between $\Delta A(t)$ and ΔZ within an industry

Industry	N	r^2	a	b (s_b)	$\pi = 0$
201	12	.0585	-.3825	2.0216 (2.5653)	
202	12	.0097	.1780	.3583 (1.1225)	
203	12	.2868	-.4728	5.6595 (2.8226)	
204	12	.1464	.0302	-3.2054 (2.4468)	
205	12	.0191	-.7102	2.0746 (4.7032)	
206	12	.0532	.2610	-2.0068 (2.6783)	
207	12	.3388	-.3905	8.7079 (3.8465)	**
208	12	.2987	-.4025	4.2776 (2.0726)	

the shift in the estimate of technological advance for the preceding two periods. These results are presented in Table 8. Three out of eleven regression coefficients differed significantly from zero with two of the coefficients being negative. In addition, eight out of eleven regression coefficients were negative.

The fourth model tests the hypothesis that the fluctuation of the rate of return within an industry is a linear

Table 8. Relationship between $\Delta A(t)$ and ΔZ in all industries with $A(t)$ lagged one period

Years	N	r^2	a	b (s_b)	$\pi = 0$
1949-50	8	.2170	.0776	-5.9126 (4.8386)	
1950-51	8	.4956	-2.1368	-7.0694 (2.9111)	*
1951-53	8	.1204	.0262	-3.9450 (4.2164)	
1953-54	8	.2302	.6078	-4.3529 (3.2497)	
1954-55	8	.0049	.7828	.9144 (5.3304)	
1955-56	8	.0001	- .1046	- .0480 (1.7958)	
1956-57	8	.1413	- .0054	-1.7296 (1.7405)	
1957-58	8	.7039	.3259	-5.1326 (1.3586)	*
1958-59	8	.1118	.3513	.5701 (.6559)	
1959-60	8	.3212	- .0525	-10.9783 (6.5152)	
1960-61	8	.7986	- .7136	9.4606 (1.9394)	*

function of the technological change parameter of the preceding two periods and these results are shown in Table 9. The regression coefficients for Industries 201 and 203 were significantly different from zero but they both have negative signs. Altogether, four of the regression coefficients were negative for this particular model.

It appears that the results fail to uncover any clear, concise pattern between observed technological change and the rate of return on total assets. However, it is reasonable to suppose, and the results suggest it, that perhaps

Table 9. Relationship between $\Delta A(t)$ and ΔZ within an industry with $A(t)$ lagged one period

Industry	N	r^2	a	b (s_b)	$\pi = 0$
201	11	.4587	.4584	-5.4496 (1.9733)	**
202	11	.0431	- .3748	.5630 (.8837)	
203	11	.4370	.8060	-6.9762 (2.6389)	**
204	11	.1691	.3032	3.4142 (2.5224)	
205	11	.0682	-1.0072	5.2738 (6.4958)	
206	11	.0783	- .4401	2.5004 (2.8593)	
207	11	.0492	.1526	-3.2174 (4.7121)	
208	11	.0217	- .3096	-1.1864 (2.6554)	

technological change has an adverse or favorable effect on rates of return depending upon the nature of the advance. That is, it may increase output of manufacturing services so that rates of return on total assets decline. The increase in the number of negative coefficients when the independent variable was lagged one period lends some small degree of credence to this approach. In light of the fact that profits or rates of return actually depend upon such a multitude of variables that the information gained from such a simple model as used here appears remarkable.

Change in Relative Shares

The results of the test of the hypothesis that the relative share of capital was unchanged between 1925 and 1958 are shown in Table 10. There were seven out of 35 b values for the four-digit industries which were significantly different from zero. In addition these seven b values were all positive. The regression coefficient for Industry 201, Meat Products, was positive but it is heavily weighted by Industry 2011, Meat Packing. Four of the regression coefficients for the four-digit industries were negative but none were significantly different from zero.

Table 10. Results of test of constant relative shares for labor

Industry number	N	r^2	b	s_b	$\pi = 0$
201	9	.4545	.0044	.0018	**
2011	10	.5817	.0057	.0017	*
2013	9	.2826	.0033	.0020	
2015	8	.3138	.0038	.0023	
202	5	.6120	.0032	.0014	
2021	10	.3629	.0036	.0017	
2022	10	.6501	.0055	.0014	*
2023	10	.0657	.0011	.0015	
2024	10	.3999	.0046	.0020	**
2025	4	.0161	- .0022	.0126	
203	4	.0395	.0010	.0033	
2031	10	.1351	.0019	.0017	
2032	4	.4193	.0138	.0115	
2033	10	.2890	.0025	.0014	
2035	4	.5469	.0032	.0021	
2037	4	.7218	- .0084	.0037	
204	10	.0339	.0009	.0018	
2041	9	.2204	.0024	.0017	
2042	9	.3156	.0026	.0015	
2043	3	.9803	.0066	.0009	
2044	8	.0001	- .0001	.0027	
205	4	.0454	.0010	.0033	
2051	10	.6891	.0064	.0015	*
2052	5	.5155	.0045	.0025	
206	3	.0242	.0005	.0029	
2062	7	.1308	- .0059	.0068	
207	4	.2597	- .0038	.0045	
2071	9	.0838	.0014	.0017	
2072	8	.0164	.0012	.0039	
208	7	.0693	.0011	.0018	
2081	6	.7434	.0100	.0029	**
2082	7	.6871	.0070	.0021	**
2083	7	.4635	.0129	.0062	
2084	5	.0082	.0007	.0044	
2085	5	.0061	.0006	.0047	
2086	3	.9063	.0123	.0040	
2087	9	.0464	.0011	.0019	

Table 10. (Continued)

Industry number	N	r^2	b	s_b	$\pi = 0$
209	5	.6927	.0013	.0005	
2092	4	.0014	.0003	.0056	
2096	5	.5298	.0030	.0016	
2096X	7	.6095	.0087	.0031	
2097	10	.6831	.0075	.0018	*
2098	8	.1978	.0026	.0022	
2099	8	.2078	.0025	.0020	

SUMMARY AND CONCLUSIONS

This chapter is provided to permit the reader to determine quickly the problem, the method of attack, and the results which were obtained from applying the statistical models to the empirical information.

The Problem

The primary problem in this study was to estimate the parameters of the production function of the Food and Kindred Products Industry. Related problems investigated were: 1. the change in factor shares over the period of the study; 2. the relationship between technological change and the rate of return on total assets.

Relevancy of the Study

The food processing industries are an important part of the marketing system which moves agricultural products from the producer to the consumer. If policy makers are to help improve the efficiency of this system, they must have some knowledge of the parameters of the production function of the industries involved.

Method of Solution

The elasticity coefficients were estimated by the use of the regression equation

$$\log (V/L) = \log a + b \log (w) \tag{72}$$

which is the locus of the profit maximizing equilibrium points given by the factor-product conditions for a firm. This study aggregated all plants, and or firms, within a state and used the state totals for the equilibrium points. The use of Equation 72, under certain assumptions, permits the researcher to make additional inferences about the production function and some of its underlying parameters. These assumptions were: 1. a linear homogeneous production function; 2. firms were profit maximizers; 3. firms were price takers in factor markets; 4. workers were paid according to their marginal value productivity; 5. that all firms have the same technological knowledge; 6. prices of output and raw material inputs do not vary systematically with wage rates; 7. the firms are in equilibrium with respect to the optimum combination of inputs.

The technological advance of the three-digit food processing industries was estimated by a procedure which divided the increase in output per worker over time into two parts, a component due to increased use of capital, and a component attributable to technological advance or the shift in the production function.

The relationship between the estimate of technological advance and the rate of return on total assets was estimated by linear regression models utilizing both lagged and non-lagged first differences of technological advance as the independent variable and the first differences of the rate of return as the independent variable.

A linear regression model utilizing time as the independent variable and the relative share of labor as the dependent variable was used to estimate the change in the relative share of labor over the period of the study.

Results

It was found that the elasticity of substitution of labor for capital was consistently equal to either zero or one, or within the interval zero and one. None of the elasticity coefficients were statistically less than zero and only nine out of a possible 372 were statistically greater than one. The results also indicated that even though the elasticity may be constant within time periods for an industry it can fluctuate between time periods. This is consistent with the model and the stability of the substitution coefficient would depend upon the nature of the technological advance between the two time periods in question.

In general, the results of estimating the elasticity of substitution produced results that are consistent with everyday observations on production techniques. That is, some substitution of labor for capital is observed in everyday experiences and seldom does a situation appear which does not permit any substitution.

Estimates of the technological change parameter appear to be reasonable to the extent that the estimates for eight different three-digit industries were grouped together with the values varying between 1.575 for Industry 202, Dairy Products, to a high of 3.307 for Industry 206, Sugar Refining. It was also found that technological advance had contributed more to productivity gains than had capital accumulation. No hypotheses were tested concerning the relationships between the measure of technological change and the elasticity of substitution parameter.

The test of the hypothesis that the rate of return on total assets was a function of technological advance did not disclose any strong relationship between the two variables.

The test of the hypothesis that the relative share of labor within a four-digit industry was constant between 1925 and 1958 could only be rejected in seven out of a possible 35 cases. There did not appear to be any relationship between the constancy of relative shares and the estimate of the elasticity of substitution of labor for capital.

Implications of the results

It would be concluded that in most of the four-digit food processing industries that substitution of capital for labor, or vice-versa, can be made over a wide range of capital-labor ratios. The exceptions, of course, are those industries where the elasticity coefficient was consistently close to or equal to zero. In those cases where the elasticity coefficient was close to or equal to one, the substitution of one factor for another can take place almost without limit as the marginal product of neither factor ever reaches zero. Hence, if capital can be substituted for labor over a wide range of capital-labor ratios then the relative prices of the factors become important and small price changes will lead to large changes in the optimum combination of the two factors.

If the substitution of capital for labor increases the efficiency of the marketing system then policy makers can encourage this substitution by minimum wage laws, low interest rates, fast tax write-offs, and other specific programs to encourage the addition of more capital equipment by the food processors. Perhaps public funds should be appropriated for research to increase the elasticity of factor substitution in those industries where the substitution possibilities are now quite limited. In addition, policy makers can influence the amount of expenditures allocated between investment on existing kinds of machinery and equipment and on research and development for technological advance. Expenditures in either direction should increase output but there should exist an optimum amount of each, however, the optimum amount on each may not be the same from the firm's and societies' viewpoint.

The estimates of technological advance indicated that the three-digit industries have achieved varying degrees of success in attaining increased efficiency between 1939 and 1961. However, since different industries achieve and adopt significant technological discoveries at different points in time, the selection of the initial year of the series is critical to the values obtained for the technological advance of an industry. Hence, the ranking of the industries according to the measure of technological advance could be altered if a few more years were included in the period under investigation.

Suggestions for Further Research

Some interesting questions which are left unanswered by this study are the following: 1. What is the relationship between the elasticity of substitution for an industry and the structure of that industry? 2. What is the relationship between the measure of technological change for an industry and the structure of that industry? 3. What is the time lag between significant engineering and management tools and their widespread adoption within an industry?

4. What has been the effect of significant wage increases on the technological advance of an industry, the number of people employed in that industry, and the output per man-year?

LITERATURE CITED

1. Douglas, Paul H. Theory of wages. New York, N. Y., The Macmillan Company. 1934.

2. Mendershausen, Horst. On the significance of Professor Douglas' production function. Econometrica 6: 143-153. 1938.

3. Brown, E. H. Phelps. The meaning of the fitted Cobb-Douglas function. The Quarterly Journal of Economics 71: 546-560. 1957.

4. Douglas, Paul H. Are there laws of production. The American Economic Review 38: 1-41. 1948.

5. Leontief, Wassily. The structure of the American economy, 1919-1929. Md ed. New York, N. Y., Oxford University Press. 1951.

6. Solter, W. E. G. Productivity and technical change. Cambridge, England, Cambridge University Press. 1960.

7. Solow, Robert M. Technical change and the aggregate production function. The Review of Economics and Statistics 39: 312-320. 1957.

8. Hicks, J. R. The theory of wages. New York, N. Y., Peter Smith Publishing Company. 1948.

9. Harrod, R. F. Towards a dynamic economics. London, England, Macmillan and Co., Ltd. 1948.

10. Massel, Benton F. Capital formation and Technological change in United States Manufacturing. The Review of Economics and Statistics 42: 182-188. 1960.

11. Allen, R. G. D. Mathematical economics. 2nd ed. London, England, Macmillan and Co., Ltd. 1959.

12. Allen, R. G. D. Mathematical analysis for economists. New York, N. Y., St. Martins Press. 1956.

13. Minhas, Bagicha Singh. An international comparison of factor costs and factor use. Amsterdam, Holland, North-Holland Publishing Co. 1963.

14. Arrow, Kenneth J., Chenery, Hollis B., Minhas, Bagicha and Solow, Robert M. Capital-labor substitution and economic efficiency. The Review of Economics and Statistics 43: 225-250. 1961.

15. Hamberg, D. Production functions, innovations, and economic growth. Journal of Political Economy 67: 238-245. 1959.

16. U. S. Bureau of the Census. Census of manufactures. Washington, D. C., U. S. Government Printing Office.

17. U. S. Bureau of the Census. Annual survey of manufactures. Washington, D. C., U. S. Government Printing Office.

18. United States Treasury Department, Internal Revenue Service. Source book of statistics of income. Washington, D. C.

19. Stigler, George J. Capital and rates of return in manufacturing industries. Princeton, N. J., Princeton University Press. 1963.

20. The President of the United States. Economic report of the President. Washington, D. C., U. S. Government Printing Office. 1965.

APPENDIX A

Listing of Industry Names and Numbers

Year	Industry number	Name
1958	201	Meat Products
1954		
1947		
1939		
1937		
1935		
1931		
1927		
1925		
1958	2011	Meat Packing
1954		Meat Packing Plants
1947		Meat Packing, Wholesale
1939		
1937		
1935		Meat Packing
1933		
1931		
1927		Slaughtering and Meat Packing
1925		
1958	2013	Prepared Meats
1954		
1947		
1939		Sausage, Prepared Meats and Other Meat Products
1937		Sausage and Sausage Casings
1935		
1931		Sausage, Meat Pudding, Headcheese, Etc. and Sausage Casings
1927		
1925		Sausage and Sausage Casings
1958	2015	Poultry Dressing, Wholesale
1954		Poultry Dressing Plants
1947		Poultry Dressing, Wholesale
1939		Poultry Dressing and Packing
1937		Poultry Dressing and Packing, Wholesale
1935		Poultry Dressing and Packing

Year	Industry number	Name
1931	2015	Poultry Killing, Dressing and Packing, Wholesale
1925		Poultry Killing and Dressing
1958	202	Dairy Products
1954		
1947		
1939		
1925		Butter, Cheese, and Condensed Milk, Group as a Whole
1958	2021	Creamery Butter
1954		
1947		
1939		
1937		Butter
1935		
1933		
1931		
1927		
1925		
1958	2022	Natural Cheese
1954		
1947		
1939		Cheese
1937		
1935		
1933		
1931		
1927		
1925		
1958	2023	Condensed and Evaporated Milk
1954		Concentrated Milk
1947		
1939		Condensed and Evaporated Milk
1937		
1935		
1933		
1931		
1927		
1925		

Year	Industry number	Name
1958	2024	Ice Creams and Ices
1954		
1947		
1939		
1937		Ice Cream
1935		
1933		
1931		
1927		
1925		—
1958	2025	Specialized Dairy Products
1954		
1947		
1939		Special Dairy Products
1958	2026	Fluid Milk
1954		
1958	2027	Fluid Milk and Other Products
1958	203	Canning, Preserving and Freezing
1954		Canned and Frozen Foods
1947		Canning, Preserving and Freezing
1939		Canning and Freezing
1958	2031	Canned Sea Foods
1954		
1947		
1939		Canned Fish, Crustacea and Mollusks
1937		Canned and Cured Fish, Crabs, Shrimps, Oysters and Clams
1935		
1933		Canned and Preserved Fish, Crabs, Shrimps, Oysters and Clams
1931		
1927		Fish, Crabs, Shrimps, Oysters and Clams
1925		
1958	2032	Cured Fish
1954		
1947		
1939		

Year	Industry number	Name
1958	2033	Canning and Preserving except Fish
1954		Canned Fruits and Vegetables
1947		Canning and Preserving except Fish
1939		Canned and Dried Fruits and Vegetables
1937		Canned and Dried Fruits and Vegetables; Canned and Bottled Juices; Preserves, Jellies, Fruit Butters, Pickles, and Sauces
1935		
1933		Canned and Dried Fruits and Vegetables; Preserves, Jellies, Fruits, Butters, Pickles and Sauces
1931		
1927		Fruits and Vegetables, Pickles, Jellies, Preserves, and Sauces
1925		
1939	2034	Preserves, Jams, Jellies and Fruit Butters
1958	2035	Pickles and Sauces
1954		
1947		
1939		Pickled Fruits and Vegetable Sauces and Seasonings
1958	2036	Fresh or Frozen Packaged Fish
1954		Packaged Seafood
1958	2037	Frozen Foods
1954		Frozen Fruits and Vegetables
1947		Frozen Foods
1939		Quick-frozen Foods
1958	204	Grain-Mill Products
1954		
1947		
1939		
1937		
1935		
1933		
1931		
1927		
1925		Flour, Feed, and Other Grain-Mill Products

Year	Industry number	Name
1958	2041	Flour and Meal
1954		
1947		
1939		Flour and Other Grain-Mill Products
1937		
1935		
1933		
1931		
1927		
1925		
1958	2042	Prepared Animal Feeds
1954		
1947		
1939		Prepared Feeds (Including Mineral) for Animals and Fowl
1937		Prepared Feeds
1935		
1933		Prepared Feeds for Animal ~~and~~ Fowl
1931		
1927		
1954	2043	Cereal Breakfast Food
1933		Ceral Preparations
1931		
1958	2044	Rice Cleaning and Polishing
1954		Rice Milling
1947		Rice Cleaning and Polishing
1939		
1937		
1935		
1931		
1927		
1925		
1958	2045	Blended and Prepared Flour
1954		
1939		Blended and Prepared Flour made from Purchased Flour
1958	205	Bakery Products
1954		
1947		
1939		

Year	Industry number	Name
1958	2051	Bread and Other Bakery Products
1954		Bread and Related Products
1947		Bread and Other Bakery Products
1939		
1937		
1935		
1933		
1931		
1927		
1925		
1954	2052	Biscuits and Crackers
1947		Biscuits, Crackers and Pretzels
1939		
1931		Biscuits and Crackers
1925		
1931	205XX	Bakery Products, other than Biscuits and Crackers
1925		
1958	206	Sugar
1954		
1947		
1939	2062	Cane Sugar Refining
1935		
1933		
1931		
1927		
1925		
1954	2063	Beet Sugar
1947		
1939		
1937		
1933		
1931		
1927		
1925		
1958	207	Candy and Related Products
1954		
1947		Confectionery and Related Products
1939		Candy and Other Related Products

Year	Industry number	Name
1958	2071	Confectionery Products
1954		
1947		
1937		
1935		
1933		
1931		
1927		
1925		
1947	2072	Chocolate and Cocoa Products
1939		
1937		
1935		
1933		
1931		
1927		
1925		
1958	2073	Chewing Gum
1954		
1947		
1925		
1958	208	Beverages
1954		
1947		
1939		
1931		
1927		
1925		
1954	2081	Bottled Soft Drinks
1947		
1939		Non-alcoholic Beverages
1937		
1935		
1933		
1958	2082	Malt Liquors
1954		Beer and Ale
1947		Malt Liquors
1939		
1937		
1935		
1933		

Year	Industry number	Name
1939	2083	Malt
1937		
1935		
1933		
1931		
1927		
1925		
1958	2084	Wines and Brandy
1947		
1939		Wines
1937		Liquors, Vinous
1935		
1954	2085	Distilled Liquors
1947		
1939		
1937		
1935		
1958	2086	Bottled and Canned Soft Drinks
1939		Liquors, Rectified and Blended
1935		
1958	2087	Flavorings
1954		
1947		
1939		Flavoring Extracts and Flavoring Sirups, n.e.c.
1937		Flavoring Extracts, Flavoring Sirups and Related Products
1935		
1931		Flavoring Extracts and Flavoring Sirups
1927		
1925		
1958	209	Miscellaneous Food Preparations
1954		Miscellaneous Foods
1947		Miscellaneous Food Preparations
1935		
1925		Food Preparations
1958	2091	Cotton-seed Oil Mills

Year	Industry number	Year
1958	2092	Soybean Oil Mills
1954		Shortening and Cooking Oils
1947		
1939		Cooking and Other Edible Fats and Oils, n.e.c.
1931	2093	Oleomargarine and Other Margarines, Not Made in Meat Packing Establishments
1927		Oleomargarine
1958	2094	Grease and Tallow
1958	2095	Animal Oils
1958	2096	Shortening and Cooking Oils
1937		Shortening, Vegetable Oils and Salad Oils
1935		Shortening, Vegetable Cooking Oils, and Salad Oils
1933		
1931		
1947	2096X	Vinegar and Cider
1939		
1937		
1935		
1931		
1927		
1925		
1958	2097	Manufactured Ice
1954		
1947		
1939		
1937		
1935		
1933		
1931		
1927		
1925		
1958	2098	Macaroni and Spaghetti
1954		
1947		

Year	Industry number	Name
1939	2098	Macaroni, Spaghetti, Vermicelli, and Noodles
1937		
1935		
1931		
1927		
1925		
1947	2099A	Liquid, Frozen and Dried Eggs
1958	2099	Food Preparations, not elsewhere classified
1954		
1947		
1939		
1937		
1935		
1933		
1931		
1927		
1925		
1927	209XX	Lard Substitutes and Vegetable Cooking Oils
1925		Lard Substitutes and Cooking Fats
1931	3000	Peanuts, Walnuts and Other Nuts, Processed or Shelled
1927		Peanuts, Walnuts and Other Nuts, Processed
1925		Peanuts; Grading, Roasting, Cleaning, Shelling
1931	2001	Coffee and Spices, Roasting and Grinding
1927		
1939	3002	Corn Sirups, Corn Sugar, Corn Oil, and Starch
1925		
1939	3003	Salad Dressings

APPENDIX B

Table 11. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
201	1958	45	.5278	*	.4088	.7633	*	**X[a]	7.873	4.305	.585
						(.1101)			(1.744)	(.856)	(.077)
	1954	46	.4416	*	.3941	.6891	*	**X[a]	6.076	3.626	.639
						(.1168)			(1.346)	(.661)	(.097)
	1947	48	.2515	*	.4372	.5373	*	*X[a]	4.615	2.605	.615
						(.1367)			(.833)	(.413)	(.009)
	1939	48	.5923	*	.2549	.7367	*	*X[a]	2.163	1.275	.591
						(.0901)			(.454)	(.256)	(.095)
	1937	38	.3526	*	.3395	.4820	*	*X[a]	2.495	1.303	.546
						(.1089)			(.444)	(.265)	(.094)
	1935	37	.3479	*	.3207	.6334	*	**X[a]	2.362	1.192	.537
						(.1466)			(.459)	(.199)	(.097)
	1931	42	.0020		.5353	.0345		*	3.610	1.425	.385
						(.1211)			(.120)	(.140)	(.079)
	1927	37	.4868	*	.2612	.9972	*		2.738	1.487	.546
						(.1731)			(.547)	(.206)	(.083)
	1925	38	.0838		.4367	.1360		*	2.904	1.434	.496
						(.0750)			(.567)	(.324)	(.078)
2011	1958	39	.4832	*	.3885	.7803	*		8.544	4.923	.618
						(.1326)			(1.598)	(.775)	(.080)
	1954	40	.2207	*	.3996	.6426	*		6.176	3.967	.682
						(.1959)			(1.401)	(.563)	(.014)
	1947	38	.9411	*	.2246	.9769	*		2.507	1.563	.641
						(.0407)			(.298)	(.146)	(.117)
	1939	42	.0558		.3256	.1431		*	2.245	1.343	.606
						(.0930			(.473)	(.297)	(.136)

[a]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 11. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	π = 0	π = 1	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2011	1937	36	.1332	**	.3560	.2760	**	*X[a]	2.479	1.340	.555
						(.1208)			(.405)	(.244)	(.106)
	1935	37	.0958		.3273	.4857		**	2.382	1.224	.542
						(.2522)			(.470)	(.160)	(.115)
	1933	35	.1923	*	.3118	.6051	*		2.160	1.065	.504
						(.2159)			(.447)	(.161)	(.109)
	1931	36	.2839	*	.4512	.8881	*		3.467	1.226	.394
						(.2419)			(.934)	(.184)	(.075)
	1927	35	.4523	*	.2336	1.1269	*		2.703	1.483	.550
						(.2158)			(.561)	(.184)	(.092)
	1925	37	.0391		.3820	.2054		*	2.720	1.392	.497
						(.1721)			(.750)	(.339)	(.084)
2013	1958	26	.3738	*	.3913	.8392	*		8.706	4.288	.539
						(.2217)			(.353)	(.903)	(.121)
	1954	30	.3440		.6576	.3313		*	7.160	3.682	.570
						(.2052)			(1.755)	(.675)	(.142)
	1947	31	.9673	*	.3706	.9426	*		5.330	2.793	.524
						(.0322)			(.750)	(.424)	(.117)
	1939	33	.2525	*	.3245	.4830	*	*X[a]	2.399	1.266	.579
						(.1493)			(.566)	(.304)	(.128)
	1937	26	.1495		.4120	.3228		*	2.841	1.302	.500
						(.1572)			(.593)	(.268)	(.094)
	1935	24	.1051		.3755	.2334		*	2.539	1.294	.517
						(.1452)			(.410)	(.295)	(.080)
	1931	29	.2041	**	.4980	.8441	**		3.996	1.270	.334
						(.3208)			(1.333)	(.202)	(.082)
	1927	22	.3473	*	.3707	.6966	*		3.297	1.604	.489
						(.2135)			(.664)	(.282)	(.086)
	1925	22	.5880	*	.2820	1.0351	*		3.210	1.622	.488
						(.1937)			(.840)	(.311)	(.077)

Table 11. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2015	1958	29	.1521	**	.4539	.6190	**		5.193	2.554	.519
						(.2813)			(1.286)	(.379)	(.115)
	1954	33	.1610	**	.3754	.6755	**		4.328	2.323	.510
						(.2770)			(1.174)	(.418)	(.227)
	1947	25	.4211	*	.2797	.9131	*		3.259	1.759	.537
						(.2232)			(.947)	(.374)	(.130)
	1939	32	.4077	*	.2628	.7459	*		1.544	.767	.507
						(.1641)			(.572)	(.248)	(.128)
	1937	20	.4259	*	.3482	.9004	*		1.950	.838	.429
						(.2464)			(.673)	(.203)	(.094)
	1935	23	.3788	*	.3490	.7690	*		1.856	.770	.437
						(.2149)			(.496)	(.154)	(.085)
	1931	18	.0892		.4854	.4992			2.898	.876	.299
						(.3989)			(.521)	(.095)	(.052)
	1925	10	.3717		.3356	.3142		*	2.160	.987	.488
						(.1444)			(.401)	(.238)	(.108)

Table 12. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
202	1958	49	.3608	*	.6328	.5420	*	*X[a]	9.542	4.344	.473
						(.1052)			(1.208)	(.611)	(.053)
	1954	31	.2950	*	.5703	.5507	*	*X[a]	7.456	3.520	.477
						(.1581)			(.861)	(.419)	(.052)
	1947	37	.5142	*	.3161	1.3100	*		6.202	2.282	.374
						(.2152)			(1.454)	(.298)	(.062)
	1939	47	.3138	*	.4158	.8386	*		3.129	1.221	.385
						(.1849)			(.733)	(.181)	(.079)
	1925	44	.3413	*	.3534	1.1436	*		2.984	1.257	.377
						(.2462)			(.643)	(.139)	(.082)
2021	1958	19	.0464		1.1699	-.4126		*	8.859	3.728	.423
						(.4534)			(2.032)	(.470)	(.157)
	1954	24	.1755	**	.4632	.6960	**		6.618	3.175	.475
						(.3216)			(1.560)	(.419)	(.110)
	1947	22	.1405		.5176	.6103			5.425	2.202	.416
						(.3375)			(1.253)	(.283)	(.182)
	1939	36	.1049		.3444	.5590			2.423	1.138	.448
						(.2800)			(.561)	(.146)	(.114)
	1937	38	.1662	**	.4069	1.4322	**		3.114	1.099	.361
						(.5346)			(1.039)	(.154)	(.077)
	1935	39	.3108	*	.4533	.7089	*		3.033	1.077	.341
						(.1735)			(.680)	(.188)	(.073)
	1933	35	.0607		.5013	.2926		*	3.185	.971	.299
						(.2003)			(.584)	(.177)	(.022)
	1931	41	.0176		.6464	.1733		*	4.596	1.144	.265
						(.2074)			(.814)	(.157)	(.055)

[a]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 12. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2021	1927	40	.0152		.4436	.4054			3.333	1.298	.368
						(.5284)			(1.036)	(.184)	(.110)
	1925	29	.5754	*	.3395	1.3295	*		3.023	1.268	.402
						(.2198)			(.518)	(.135)	(.060)
2022	1958	9	.0686		.5954	.5159			8.016	3.788	.434
						(.7183)			(1.993)	(.482)	(.138)
	1954	18	.2157		.4050	.7888			6.554	3.224	.481
						(.3760)			(1.958)	(.478)	(.092)
	1947	17	.1549		.2095	1.3097			4.758	2.183	.418
						(.7898)			(1.721)	(.249)	(.123)
	1939	31	.0152		.4164	.1363		*	2.689	1.072	.448
						(.2039)			(.589)	(.227)	(.137)
	1937	26	.0986		.4531	.4558			2.974	1.041	.308
						(.2813)			(.812)	(.203)	(.119)
	1935	20	.5611	*	.4434	.9188	*		2.900	1.034	.330
						(.1915)			(.775)	(.207)	(.062)
	1933	20	.0498		.4699	.3577			2.964	.931	.309
						(.3682)			(.825)	(.154)	(.011)
	1931	27	.0700		.5655	.3898		**	3.997	1.183	.255
						(.2842)			(.842)	(.168)	(.073)
	1927	20	.5217	*	.4239	.8031	*		3.335	1.297	.321
						(.1812)			(1.062)	(.344)	(.097)
	1925	18	.3868	*	.3781	.9388	*		2.797	1.131	.320
						(.2955)			(.992)	(.284)	(.157)
2023	1958	14	.0935		.2571	1.3992			15.654	4.536	.313
						(1.2579)			(4.687)	(.328)	(.100)
	1954	17	.0004		1.1483	-.0357		**	13.822	3.653	.303
						(.4729)			(3.464)	(.433)	(.073)

Table 12. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2023	1947	16	.1946		.6142	.7975 (.4336)			8.936 (1.656)	2.605 (.266)	.300 (.052)
	1939	23	.3360	*	.4846	.9719 (.2981)	*		3.842 (1.261)	1.225 (.227)	.344 (.080)
	1937	23	.0038		.5808	.0910 (.3194)		*	3.957 (.861)	1.203 (.197)	.300 (.081)
	1935	21	.2143	**	.5598	.9220 (.4051)	**		4.126 (1.177)	1.118 (.158)	.269 (.072)
	1933	19	.0280		.6905	-.6067 (.8675)			4.921 (1.230)	1.047 (.073)	.226 (.063)
	1931	19	.0220		.8203	-.2854 (.4610)		**	6.496 (1.636)	1.183 (.156)	.185 (.051)
	1927	17	.0146		.5952	.3702 (.7840)			4.611 (1.830)	1.316 (.144)	.291 (.088)
	1925	17	.2140		.3116	2.3039 (1.1400)			3.568 (1.322)	1.246 (.081)	.341 (.107)
2024	1958	32	.3613	*	.4000	1.0249 (.2488)	*		11.089 (3.215)	4.155 (.689)	.390 (.086)
	1954	38	.2506	*	.5005	.7761 (.2237)	*		8.393 (1.729)	3.445 (.490)	.409 (.085)
	1947	25	.5850	*	.4021	.9738 (.1710)	*		5.988 (1.268)	2.408 (.400)	.402 (.053)
	1939	46	.3252	*	.4583	.6988 (.1517)	*		3.468 (.760)	1.286 (.227)	.362 (.072)
	1937	45	.3742	*	.5406	1.2575 (.2480)	*		4.776 (1.309)	1.264 (.161)	.234 (.058)
	1935	45	.2298	*	.5202	.6784 (.1894)	*		3.929 (.744)	1.264 (.160)	.292 (.056)
	1933	47	.3528	*	.4882	1.4080 (.2843)	*		3.941 (1.212)	1.166 (.145)	.263 (.068)

Table 12. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2024	1931	47	.2548	*	.7686	.7951	*		7.476	1.324	.161
						(.2027)			(1.870)	(.186)	(.033)
	1927	48	.2567	*	.5389	.7190	*		4.764	1.534	.322
						(.1804)			(.982)	(.221)	(.028)
	1925	48	.0816	**	.4859	.7137	**		4.169	1.452	.328
						(.3531)			(1.098)	(.231)	(.047)
2025	1958	3	.2614		2.7435	- 3.4356			7.820	3.544	.575
						(5.7750)			(3.897)	(.260)	(.272)
	1954	5	.6154		-1.7220	4.9286			12.191	3.588	.248
						(2.2494)			(9.783)	(.363)	(.165)
	1947	4	.4716		.2662	1.4560			6.590	2.351	.370
						(1.0897)			(2.112)	(.352)	(.088)
	1939	3	.8639		.1540	1.9463			3.325	1.469	.529
						(.7726)			(2.494)	(.502)	(.199)
2026	1958	39	.2449	*	.5362	.6446	*		9.226	4.570	.508
						(.1861)			(1.526)	(.554)	(.072)
	1954	28	.2320	*	.3736	.7905	*		6.124	3.214	.481
						(.2821)			(1.982)	(.522)	(.138)
2027	1954	49	.2859	*	.4848	.6602	*	**X[a]	7.403	3.779	.522
						(.1522)			(1.269)	(.493)	(.070)

Table 13. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
203	1958	31	.4417	*	.2885	1.1667	*		7.266	2.993	.390
						(.2435)			(2.579)	(.662)	(.212)
	1954	31	.6974	*	.2602	1.1589	*		5.444	2.514	.432
						(.1418)			(2.018)	(.641)	(.094)
	1947	31	.8315	*	.3499	.9104	*		3.611	1.683	.461
						(.0761)			(1.180)	(.552)	(.065)
	1939	35	.7690	*	.3895	1.0043	*		1.573	.623	.372
						(.0958)			(.814)	(.224)	(.096)
2031	1958	9	.5619	**	.3736	.8682	**		6.739	3.246	.470
						(.2898)			(2.660)	(1.037)	(.108)
	1954	11	.9120	*	.0912	1.4245	*	**Y[a]	3.669	2.090	.523
						(.1475)			(1.792)	(.701)	(.154)
	1947	8	.8612	*	.2694	1.3544	*		3.788	1.603	.367
						(.2220)			(2.207)	(.617)	(.145)
	1939	15	.9449	*	.4724	1.2024	*	**Y[a]	1.362	.506	.365
						(.0805)			(1.138)	(.403)	(.130)
	1937	16	.8464	*	.4208	.9866	*		1.696	.624	.368
						(.1123)			(1.299)	(.468)	(.117)
	1935	12	.8622	*	.3303	.9346	*		1.562	.704	.386
						(.1182)			(.951)	(.431)	(.115)
	1933	14	.7782	*	.3650	.9041	*		1.492	.603	.374
						(.1394)			(.791)	(.330)	(.114)
	1931	18	.8838	*	.4519	1.1463	*		2.211	.766	.371
						(.1039)			(1.704)	(.492)	(.122)
	1927	17	.9405	*	.3329	1.0812	*		1.979	.889	.478
						(.0702)			(1.570)	(.615)	(.096)

[a]Y in the column entitled $\pi = 1$ denotes that π is greater than one.

Table 13. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2031	1925	14	.5410	*	.1582	.7891	*		1.613	1.304	.429
						(.2098)			(1.234)	(2.257)	(.030)
2032	1958	4	.7946		.0628	1.6988			11.223	3.712	.291
						(.6108)			(5.671)	(.858)	(.083)
	1954	4	.6451		.2522	.8399			5.110	3.438	.626
						(.4405)			(1.793)	(.963)	(.145)
	1947	3	.9209		-.2801	2.2282			5.414	2.816	.475
						(.6530)			(2.011)	(.432)	(.114)
	1939	10	.6572	*	.6827	1.0820	*		5.133	.967	.238
						(.2763)			(3.278)	(.279)	(.149)
2033	1958	21	.7298	*	.2072	1.3845	*		7.131	2.873	.383
						(.1933)			(2.369)	(.584)	(.085)
	1954	30	.8728	*	.1170	1.4821	*	*Y[a]	5.618	2.619	.426
						(.1069)			(2.146)	(.655)	(.101)
	1947	30	.8318	*	.2901	1.1084	*		3.778	1.793	.474
						(.0942)			(1.292)	(.526)	(.077)
	1939	31	.8394	*	.4366	1.2398	*	**Y[a]	1.468	.572	.359
						(.1007)			(.976)	(.220)	(.105)
	1937	37	.6109	*	.3875	.7825	*	**X[b]	1.719	.616	.390
						(.1056)			(.818)	(.244)	(.085)
	1935	37	.7400	*	.4307	1.0570	*		1.685	.622	.373
						(.1059)			(.774)	(.216)	(.088)
	1933	37	.7069	*	.5312	1.1513	*		1.718	.532	.320
						(.1253)			(.847)	(.192)	(.107)
	1931	37	.5543	*	.5688	1.1132	*		2.118	.560	.283
						(.1687)			(1.324)	(.190)	(.102)

[b]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 13. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L $(s_{V/L})$	W/L $(s_{W/L})$	W/V $(s_{W/V})$
2033	1927	36	.0239		.2586	.1235 (.1352)		*	1.992 (.993)	1.120 (1.154)	.376 (.074)
	1925	35	.8259	*	.4322	1.3253 (.1059)	*	*Y[a]	2.095 (.928)	.803 (.251)	.371 (.094)
2035	1958	21	.4048	*	.2094	1.2874 (.3581)	*		8.539 (4.279)	3.468 (.667)	.430 (.114)
	1954	20	.1828		.4490	.7497 (.3736)			6.504 (2.642)	2.846 (.580)	.465 (.134)
	1947	21	.0742		.5109	.5967 (.4836)			5.395 (2.606)	2.069 (.380)	.389 (.145)
	1939	21	.2932	**	.3325	.6960 (.2479)	**		2.137 (.853)	.936 (.236)	.391 (.145)
2036	1958	14	.6733	*	.2587	1.0870 (.2186)	*		4.978 (2.537)	2.423 (.902)	.496 (.201)
	1954	11	.3969	**	.3262	.6902 (.2836)	**		3.818 (1.471)	2.258 (.816)	.601 (.224)
2037	1958	11	.2891		.4775	.8419 (.4400)			8.071 (2.162)	3.163 (.518)	.375 (.081)
	1954	12	.5027	*	.2335	1.2666 (.3983)	*		5.387 (2.230)	2.380 (.533)	.381 (.146)
	1947	13	.5796	*	.3799	.5513 (.1416)	*	*X[b]	2.927 (.995)	1.434 (.618)	.533 (.140)
	1939	3	.3524		.2824	.4449 (.6032)			1.595 (.207)	.660 (.121)	.508 (.061)

Table 14. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
204	1958	37	.4575	*	.1352	1.5251	*		14.215	4.535	.309
						(.2807)			(4.512)	(.676)	(.094)
	1954	43	.5687	*	.2514	1.3094	*		10.228	3.748	.356
						(.2351)			(2.939)	(.548)	(.092)
	1947	41	.4512	*	.3983	1.2006			8.193	2.625	.310
						(.1743)			(2.055)	(.394)	(.058)
	1939	43	.6269	*	.4319	1.1012	*		3.410	1.216	.334
						(.1327)			(.942)	(.230)	(.066)
	1937	39	.6726	*	.4505	1.1212	*		3.550	1.210	.326
						(.1286)			(.977)	(.241)	(.061)
	1935	37	.2642	*	.4856	.7726	*		3.516	1.162	.307
						(.2179)			(.900)	(.191)	(.094)
	1933	44	.4360	*	.5342	.2908	*	*Y[a]	3.598	1.003	.255
						(.2265)			(1.362)	(.178)	(.091)
	1931	44	.3253	*	.6433	1.0888	*		5.190	1.120	.171
						(.2418)			(1.808)	(.193)	(.073)
	1927	48	.3649	*	.4123	1.0475	*		3.798	1.409	.335
						(.2037)			(1.084)	(.221)	(.093)
	1925	48	.4989	*	.3698	1.2526	*		3.608	1.385	.363
						(.1851)			(.979)	(.212)	(.089)
2041	1958	20	.4618	*	.3073	1.2059	*		14.230	4.789	.348
						(.3069)			(6.355)	(.983)	(.122)
	1954	21	.3070	*	.1507	1.4563	*		11.519	4.068	.379
						(.5020)			(4.300)	(.575)	(.129)
	1947	18	.1075		.6914	.6752			10.850	3.103	.275
						(.4864)			(2.935)	(.366)	(.071)

[a]Y in the column entitled $\pi = 1$ denotes that π is greater than one.

Table 14. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2041	1939	39	.6180	*	.4008	1.1420	*		3.379	1.259	.343
						(.1476)			(1.251)	(.304)	(.097)
	1937	35	.6203	*	.4014	1.4050	*	**Y[a]	3.629	1.269	.329
						(.1913)			(1.166)	(.232)	(.085)
	1935	37	.3725	*	.4707	.9449	*		3.528	1.171	.306
						(.2073)			(1.062)	(.200)	(.092)
	1933	42	.3830	*	.5099	1.4398	*		3.641	1.039	.283
						(.2890)			(1.585)	(.172)	(.106)
	1931	44	.4706	*	.6009	1.3788	*		4.820	1.111	.199
						(.2256)			(1.742)	(.194)	(.074)
	1927	46	.2980	*	.4045	.9709	*		3.686	1.422	.340
						(.2247)			(1.119)	(.232)	(.109)
2042	1958	34	.1961	*	.5567	.8712	*		12.963	4.207	.316
						(.3118)			(3.744)	(.576)	(.081)
	1954	33	.3188	*	.2831	1.2026	*		9.232	3.587	.383
						(.3157)			(2.949)	(.522)	(.103)
	1947	22	.2711	**	.5858	.6526	**		6.874	2.390	.353
						(.2393)			(1.338)	(.366)	(.060)
	1939	38	.3869	*	.4739	.9301	*		3.538	1.171	.320
						(.1951)			(1.057)	(.222)	(.082)
	1937	39	.6654	*	.4977	.8217	*	*X[b]	3.603	1.150	.319
						(.0602)			(1.198)	(.266)	(.089)
	1935	36	.5898	*	.4857	.8756	*	**X[b]	3.327	1.154	.316
						(.0570)			(.985)	(.547)	(.076)
	1933	39	.4821	*	.4918	1.0870	*		2.877	.904	.293
						(.1852)			(.962)	(.196)	(.106)

[b]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 14. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2042	1931	39	.0608		.6536	.3904		**	4.854	1.068	.199
						(.2523)			(1.613)	(.216)	(.074)
	1927	33	.2900	*	.4349	1.1169	*		4.171	1.404	.317
						(.3139)			(1.462)	(.246)	(.108)
2043	1954	3	.2010		- 1.1515	3.7099			16.754	4.304	.287
						(7.3966)			(6.878)	(.220)	(.110)
	1933	8	.6546	*	.7618	2.5051	**		7.142	1.061	.162
						(.7428)			(3.043)	(.153)	(.061)
	1931	11	.4743	**	.7006	3.2801	**		10.157	1.187	.125
						(1.1511)			(5.948)	(.160)	(.086)
2044	1958	3	.9998	*	.4302	1.2413	*	*Y[a]	15.866	4.140	.266
						(.0196)			(6.199)	(1.311)	(.021)
	1954	3	.1788		.5762	.8430			12.128	3.697	.326
						(1.8065)			(5.913)	(.940)	(.179)
	1947	3	.9226		.4820	1.4252			13.753	2.836	.223
						(.4127)			(6.111)	(.807)	(.038)
	1939	4	.3176		.7120	1.2442			8.311	1.450	.361
						(1.2896)			(1.989)	(.160)	(.033)
	1937	4	.9804	*	.4898	1.7325	*	**Y[a]	3.580	1.054	.323
						(.1731)			(2.155)	(.352)	(.077)
	1935	4	.9078	*	.5662	1.8536	**		4.463	1.084	.269
						(.4177)			(2.182)	(.247)	(.056)
	1931	4	.7530		.8460	1.5504			6.500	.920	.149
						(.6278)			(3.645)	(.258)	(.042)
	1927	4	.3802		.4650	.8617			3.738	1.310	.360
						(.7779)			(1.032)	(.233)	(.073)
	1925	3	.0377		.6042	- .0609			3.965	1.328	.290
						(.3077)			(.323)	(.339)	(.103)

Table 14. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2045	1958	3	.0358		1.9460	- .7380			23.427	6.240	.259
						(3.8277)			(6.203)	(.414)	(.081)
	1954	3	.8792		- .6518	2.7308			16.036	4.731	.267
						(1.0119)			(5.608)	(.618)	(.088)
	1939	4	.8625		.4516	1.0871			3.304	1.131	.361
						(.3070)			(1.679)	(.440)	(.061)

Table 15. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
205	1958	12	.7924	*	.1576	1.2354	*		8.300	4.110	.495
						(.2000)			(1.774)	(.619)	(.050)
	1954	28	.4894	*	.3376	.8379	*		6.584	3.731	.568
						(.1678)			(.965)	(.446)	(.055)
	1947	36	.2988	*	.4413	.5612	*	*X[a]	4.714	2.578	.557
						(.1474)			(.547)	(.286)	(.056)
	1939	49	.6741	*	.3472	.6418	*	*X[a]	2.504	1.203	.491
						(.0651)			(.304)	(.184)	(.040)
2051	1958	49	.4396	*	.4096	.7883	*		8.095	4.267	.547
						(.1298)			(1.167)	(.497)	(.057)
	1954	47	.2580	*	.6856	.2054	*	*X[a]	6.349	3.694	.618
						(.0519)			(.780)	(.648)	(.087)
	1947	28	.3482	*	.4335	.5427	*	*X[a]	4.684	2.722	.607
						(.1456)			(.558)	(.330)	(.060)
	1939	50	.1474	*	.2964	.9550	*		2.454	1.218	.514
						(.3315)			(.420)	(.182)	(.044)
	1937	49	.6820	*	.3460	.7549	*	*X[a]	2.551	1.202	.486
						(.0752)			(.398)	(.200)	(.041)
	1935	49	.3868	*	.3258	.4517	*	*X[a]	2.212	1.099	.509
						(.0830)			(.244)	(.161)	(.053)
	1933	41	.6885	*	.3436	.8416	*		2.226	1.009	.451
						(.0906)			(.326)	(.150)	(.039)
	1931	49	.4963	*	.4635	.7530	*	**X[a]	3.353	1.202	.350
						(.1107)			(.546)	(.191)	(.044)
	1927	49	.4249	*	.4320	.6478	*	*X[a]	3.338	1.378	.403
						(.1099)			(.454)	(.188)	(.045)

[a]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 15. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2051	1925	49	.1542	*	.4370	.3056	*	*X[a]	3.031	1.380	.428
						(.1044)			(.408)	(.271)	(.088)
2052	1954	16	.5156	*	-.2034	2.1582	*		8.231	3.222	.361
						(.5591)			(2.908)	(.393)	(.147)
	1947	14	.0010		.7100	.0551			5.440	2.394	.431
						(.5117)			(.826)	(.213)	(.087)
	1939	22	.2512	**	.3855	1.1794	**		2.994	1.169	.354
						(.4553)			(.749)	(.125)	(.101)
	1931	17	.6547	*	.6861	1.1626	*		4.251	.883	.191
						(.2180)			(1.024)	(.148)	(.034)
	1925	20	.1402		.5148	.5768			3.485	1.089	.300
						(.3366)			(.630)	(.139)	(.069)
205XX	1931	31	.5940	*	.4340	.7830	*		3.292	1.273	.391
						(.1202)			(.515)	(.194)	(.038)
	1925	34	.2289	*	.4361	.2056	*	*X[a]	2.981	1.565	.467
						(.0667)			(.354)	(.765)	(.234)

Table 16. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
206	1958	5	.3057		.7574	.4267			10.540	4.780	.454
						(.3713)			(1.722)	(.946)	(.079)
	1954	5	.1295		.4043	.8068			7.634	3.794	.482
						(1.2074)			(1.993)	(.478)	(.156)
2062	1939	3	.9330		.4689	.7169			3.327	1.272	.382
						(.1922)			(.352)	(.362)	(.049)
	1937	3	.9067		.5424	.2227			3.610	1.201	.322
						(.0714)			(.372)	(.035)	(.081)
	1935	3	.5971		.4004	.3406			2.528	1.027	.419
						(.2798)			(.342)	(.280)	(.086)
	1933	3	.9786		.6086	.2962		**	4.135	1.088	.273
						(.0438)			(.392)	(.319)	(.033)
	1931	3	.9328		.6229	.6111			4.819	1.268	.362
						(.1641)			(.977)	(.376)	(.035)
	1927	3	.9230		.2870	.7699			2.518	1.412	.488
						(.2224)			(.556)	(.364)	(.050)
	1925	3	.0462		.4482	.0993			2.919	1.433	.421
						(.4511)			(.400)	(.377)	(.142)
2063	1954	4	.5578		- 1.8597	4.7102			7.929	3.825	.459
						(2.9652)			(2.303)	(.208)	(.175)
	1947	5	.7055		- .2249	2.5515			7.600	2.681	.338
						(.9518)			(2.159)	(.291)	(.105)
	1939	6	.3121		.4039	1.6565			3.954	1.271	.332
						(1.2296)			(1.552)	(.164)	(.120)
	1937	6	.2702		.2108	2.1838			2.920	1.248	.393
						(1.7943)			(1.458)	(.170)	(.305)
	1933	5	.2528		.5363	1.4792			3.953	1.060	.256
						(1.4682)			(1.557)	(.136)	(.110)

Table 16. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2063	1931	4	.0417		.6078	-.2118			3.987	1.178	.323
						(.7185)			(.790)	(.235)	(.085)
	1927	5	.4013		-.4552	4.9382			2.786	1.514	.491
						(3.4824)			(.716)	(.052)	(.145)
	1925	5	.0941		.9332	-1.8294			4.105	1.513	.337
						(3.2767)			(.897)	(.057)	(.097)

Table 17. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
207	1958	25	.5561	*	.1675	1.3009	*		7.019	3.252	.399
						(.2423)			(2.241)	(.605)	(.114)
	1954	27	.3138	*	.1929	1.2141	*		5.718	2.794	.425
						(.3590)			(2.253)	(.477)	(.160)
	1947	8	.6170	**	.2790	1.3896	**		6.134	2.296	.349
						(.4470)			(1.578)	(.332)	(.065)
	1939	36	.0788		.2545	.3868		**	2.178	1.075	.497
						(.2268)			(.600)	(.233)	(.135)
2071	1958	22	.4689	*	.2547	1.1019	*		6.545	3.166	.462
						(.2622)			(1.827)	(.585)	(.111)
	1954	24	.2859	*	.2086	1.1707	*		5.449	2.709	.473
						(.3945)			(2.109)	(.441)	(.151)
	1947	21	.6634	*	.2774	1.1800	*		4.041	1.876	.443
						(.1928)			(1.062)	(.370)	(.084)
	1937	32	.3040	*	.3368	1.0870	*		1.821	.828	.447
						(.3002)			(.536)	(.116)	(.105)
	1935	33	.3909	*	.2923	.9046	*		1.662	.823	.487
						(.2028)			(.321)	(.110)	(.078)
	1933	34	.4278	*	.3171	.9698	*		1.482	.695	.416
						(.1983)			(.361)	(.106)	(.085)
	1931	39	.1934	*	.4437	.6031	*		2.396	.760	.302
						(.2025)			(.535)	(.143)	(.093)
	1927	42	.4944	*	.2938	1.0182	*		2.055	1.033	.470
						(.1628)			(.424)	(.152)	(.089)
	1925	40	.0250		.2918	.1601		*	2.068	1.191	.448
						(.1622)			(.512)	(.241)	(.077)

Table 17. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2072	1947	3	.6390		.4077	1.2837 (.9650)			10.976 (2.815)	3.085 (.463)	.294 (.044)
	1939	4	.7337		.2213	1.3176 (.5613)			3.327 (1.014)	1.674 (.328)	.366 (.085)
	1937	5	.5395		.4693	.7409 (.3952)			3.271 (.822)	1.142 (.313)	.369 (.061)
	1935	5	.0688		.4594	.2612 (.5546)			3.048 (.468)	1.209 (.210)	.389 (.078)
	1933	6	.0011		.5432	- .1160 (1.7161)			3.704 (1.444)	1.066 (.125)	.288 (.137)
	1931	6	.0597		.6002	3.1834 (6.3144)			8.805 (7.981)	1.191 (.064)	.178 (.104)
	1927	4	.3141		.3274	2.1360 (2.2325)			4.225 (1.064)	1.367 (.086)	.313 (.066)
	1925	7	.0170		.4968	.8455 (2.8797)			4.421 (1.972)	1.385 (.078)	.342 (.117)
2073	1925	3	.1805		.4819	1.6399 (3.4944)			5.420 (5.473)	1.188 (.278)	.331 (.232)

Table 18. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
208	1958	45	.6379	*	.1706	1.3459 (.1546)	*	**Y[a]	11.320 (3.703)	4.443 (.828)	.358 (.075)
	1954	48	.7629	*	.0566	1.5229 (.1252)	*	*Y[a]	8.818 (3.161)	3.757 (.740)	.388 (.086)
	1947	42	.3209	*	.3171	1.2123 (.2788)	*		7.461 (2.707)	2.771 (.436)	.332 (.089)
	1939	49	.6221	*	.4454	.9725 (.1105)	*		4.042 (.982)	1.449 (.278)	.346 (.055)
	1931	49	.0000		.7016	.0031 (.0794)		*	5.125 (.996)	1.394 (1.439)	.220 (.052)
	1927	49	.2427	*	.5030	.5276 (.1360)	*	*X[b]	3.851 (.754)	1.408 (.255)	.362 (.070)
	1925	49	.1624	*	.5091	.3230 (.1070)	*		3.604 (.711)	1.360 (.275)	.377 (.077)
2081	1954	25	.6696	*	.3320	.9345 (.1369)	*		6.590 (1.303)	3.468 (.560)	.508 (.051)
	1947	27	.2931	*	.5623	.3973 (.1234)	*	*X[b]	5.187 (.471)	2.416 (.296)	.465 (.048)
	1939	49	.2820	*	.4818	.5997 (.1396)	*	*X[b]	3.644 (.493)	1.348 (.174)	.363 (.049)
	1937	49	.0696	*	.5868	.3664 (.1954)		*	4.321 (.918)	1.293 (.203)	.282 (.077)
	1935	48	.1618	*	.5282	.5924 (.1988)	*	**X[b]	3.968 (.965)	1.267 (.227)	.404 (.087)

[a]Y in the column entitled $\pi = 1$ denotes that π is greater than one.

[b]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 18. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2081	1933	46	.0000		.5177	.0102		*	3.423	1.002	.281
						(.2969)			(.912)	(.158)	(.081)
2082	1958	14	.5582	*	.1597	1.2960	*		15.006	6.041	.406
						(.3328)			(2.912)	(.626)	(.051)
	1954	17	.2387	**	.3843	1.0454	**		13.175	4.999	.385
						(.4820)			(2.219)	(.433)	(.081)
	1947	17	.3785	*	.4435	.9670	*		9.247	3.433	.365
						(.3199)			(1.743)	(.381)	(.003)
	1939	26	.3480	*	.4675	.9526	*		5.376	1.855	.324
						(.2662)			(1.219)	(.280)	(.073)
	1937	27	.1553	**	.5185	.7362	**		5.092	1.731	.314
						(.3434)			(1.453)	(.256)	(.098)
	1935	28	.4426	*	.5270	.9224	*		5.378	1.638	.296
						(.2030)			(1.236)	(.259)	(.055)
	1933	21	.5159	*	.7960	.9867	*		8.194	1.298	.157
						(.2192)			(1.909)	(.236)	(.001)
2083	1939	5	.2448		1.1168	- .3519		**	9.783	2.396	.216
						(.3568)			(1.433)	(.516)	(.094)
	1937	5	.0057		1.0464	- .0767			10.606	2.371	.230
						(.5846)			(2.081)	(.464)	(.062)
	1935	5	.0117		1.1079	- .1275			11.908	2.276	.191
						(.6772)			(2.936)	(.509)	(.098)
	1933	3	.1108		.6568	1.1486			8.238	1.619	.215
						(3.2544)			(3.258)	(.167)	(.069)
	1931	3	.3978		1.5841	- 3.2095			9.616	1.551	.163
						(3.9490)			(2.533)	(.078)	(.063)
	1927	3	.5946		.2734	1.2089			5.110	2.286	.439
						(.9981)			(.601)	(.181)	(.355)

Table 18. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2083	1925	3	.3319		.7824	.0503		**	6.076	1.496	.338
						(.0714)			(.763)	(.157)	(.006)
2084	1958	3	.5884		.7822	.6718			17.050	4.657	.251
						(.5618)			(3.549)	(1.159)	(.045)
	1947	6	.3602		.0219	1.9512			9.229	2.953	.225
						(1.3004)			(3.782)	(.385)	(.116)
	1939	8	.0613		.4592	.4370			3.267	1.259	.340
						(.6979)			(.824)	(.191)	(.116)
	1937	6	.6133		.4135	1.9735			3.892	1.205	.250
						(.7835)			(1.268)	(.178)	(.011)
	1935	5	.0409		.6937	.1926			5.227	1.136	.171
						(.5383)			(1.726)	(.306)	(.086)
2085	1954	4	.4652		.6239	.9762			17.163	4.186	.251
						(.7401)			(3.516)	(.623)	(.040)
	1947	7	.1014		1.6916	- 1.3127			13.712	2.758	.162
						(1.7481)			(4.386)	(.244)	(.011)
	1939	8	.1616		.5525	.5801			4.630	1.468	.298
						(.5393)			(1.507)	(.433)	(.118)
	1937	9	.4814	**	.4073	1.7038	**		4.670	1.350	.257
						(.6684)			(2.250)	(.304)	(.191)
	1935	10	.0099		.7170	.2730			6.257	1.258	.148
						(.9666)			(3.389)	(.255)	(.146)
2086	1958	29	.8236	*	.4102	.8520	*		8.469	4.047	.476
						(.0759)			(1.505)	(.739)	(.001)
	1939	10	.0030		.5973	- .0817			4.051	1.562	.304
						(.5311)			(1.415)	(.439)	(.197)
	1935	14	.0138		.6809	- .2210		**	4.993	1.014	.159
						(.5383)			(1.391)	(.167)	(.064)

Table 18. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2087	1958	12	.2091		- .0735	1.8967			19.942	5.020	.218
						(1.1664)			(10.125)	(.593)	(.130)
	1954	15	.3444	**	- .4927	2.8087	**		20.289	4.046	.216
						(1.0748)			(15.761)	(.583)	(.168)
	1947	11	.3390		- .0663	2.1686			11.523	3.109	.256
						(1.0093)			(6.802)	(.462)	(.166)
	1939	19	.0632		.5650	.9128			7.218	1.644	.137
						(.8521)			(5.630)	(.311)	(.187)
	1937	18	.2307	**	.5234	1.1518	**		5.934	1.466	.116
						(.5259)			(4.318)	(.331)	(.131)
	1935	19	.2617	**	.4281	1.8843	**		8.818	1.460	.173
						(.7676)			(4.767)	(.308)	(.143)
	1931	19	.0898		.9877	- .5979		*	10.232	1.178	.097
						(.4616)			(6.487)	(.278)	(.074)
	1927	23	.3127	*	.5187	1.1731	*		7.122	1.708	.254
						(.3795)			(4.792)	(.414)	(.137)
	1925	24	.0830		.6372	.8710			8.998	1.658	.223
						(.6173)			(4.697)	(.385)	(.147)

Table 19. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
209	1958	43	.5631	*	.2094	1.3700	*		11.185	4.017	.337
						(.1885)			(3.407)	(.681)	(.081)
	1954	28	.3838	*	.1771	1.5393	*		10.165	3.310	.301
						(.3825)			(4.641)	(.550)	(.117)
	1947	37	.3438	*	.3689	1.1538	*		6.402	2.307	.317
						(.2694)			(2.445)	(.395)	(.080)
	1935	28	.0613		.4379	.2323			2.886	.975	.301
						(.1783)			(1.134)	(.236)	(.116)
	1925	36	.5514	*	.4584	1.1670	*		3.946	1.254	.278
						(.1805)			(1.771)	(.337)	(.096)
2091	1958	9	.4657	**	- .2020	1.7390	**		6.585	3.670	.488
						(.7040)			(4.084)	(.752)	(.227)
2092	1958	7	.3985		.5368	.9729			16.949	5.030	.296
						(.5345)			(4.673)	(.927)	(.091)
	1954	7	.4953		3.0799	- 2.7874		**	26.091	4.308	.207
						(1.2584)			(2.805)	(.628)	(.113)
	1947	4	.0102		1.2823	.0875			21.339	3.366	.157
						(.6086)			(1.458)	(.269)	(.015)
	1939	7	.7480	**	.3446	1.8940	**		4.469	1.416	.285
						(.4916)			(1.753)	(.295)	(.118)
2093	1931	3	.8787		.6075	1.8269			6.368	1.276	.179
						(.6788)			(1.254)	(.127)	(.022)
	1927	3	.5780		.4398	.9743			4.604	1.687	.292
						(.8324)			(.881)	(.240)	(.002)
2094	1958	24	.1146		.8574	.1759		*	9.884	6.585	.474
						(.1043)			(2.715)	(.104)	(.106)

Table 19. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2095	1958	5	.2563		.7042	.5402			10.755	3.835	.331
						(.5313)			(3.860)	(1.178)	(.146)
209XX	1927	3	.7980		.5231	1.3616			4.000	1.127	.344
						(.6851)			(1.403)	(.284)	(.057)
	1925	3	.1871		.6776	-.8430			5.213	.983	.220
						(1.7570)			(2.385)	(.224)	(.108)
2096	1958	4	.9083	**	-.2506	2.0106	**		16.211	5.301	.319
						(.4516)			(3.239)	(.547)	(.044)
	1937	5	.9296	*	.5720	.7654	*		4.318	1.215	.231
						(.1216)			(.773)	(.280)	(.021)
	1935	4	.5415		.7268	.3396			5.341	1.011	.196
						(.2210)			(.475)	(.200)	(.027)
	1933	4	.0210		.5516	.1810			3.571	.934	.259
						(.8746)			(.736)	(.172)	(.066)
	1931	4	.1648		.6632	.5666			4.917	1.020	.259
						(.9020)			(2.218)	(.336)	(.102)
2096X	1947	3	.7905		1.0262	-.9460			4.595	2.470	.539
						(.4870)			(.750)	(.406)	(.190)
	1939	14	.0983		.3516	.3204		**	2.306	1.007	.419
						(.2800)			(.631)	(.268)	(.146)
	1937	8	.3126		.3366	.8291			2.525	1.165	.466
						(.5020)			(.732)	(.241)	(.138)
	1935	9	.5360	**	.3581	1.0493	**		2.606	1.156	.400
						(.3690)			(1.058)	(.296)	(.102)
	1931	12	.0039		.5579	.1793			4.125	1.126	.273
						(.9006)			(2.217)	(.196)	(.137)

Table 19. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2096X	1927	14	.3228	**	.4185	.7996	**		3.679	1.460	.392
						(.3343)			(1.285)	(.352)	(.132)
	1925	12	.0909		.5792	.1681		*	4.103	1.286	.349
						(.1681)			(1.801)	(.284)	(.080)
2097	1958	25	.6485	*	.1552	1.2680	*		7.109	3.477	.487
						(.1946)			(2.297)	(.781)	(.120)
	1954	27	.6988	*	.2251	1.0764	*		5.676	3.063	.538
						(.1413)			(1.542)	(.649)	(.087)
	1947	35	.4577	*	.4129	.7602	*		4.900	2.289	.461
						(.1440)			(1.071)	(.407)	(.073)
	1939	45	.4250	*	.3610	.9116	*		2.785	1.204	.422
						(.1617)			(.857)	(.238)	(.098)
	1937	45	.1876	*	.5503	.5218	*	*X[a]	4.178	1.321	.309
						(.1656)			(1.003)	(.249)	(.075)
	1935	42	.2623	*	.5007	.5859	*	**X[a]	3.842	1.350	.348
						(.1554)			(1.004)	(.305)	(.075)
	1933	44	.4198	*	.5041	1.1491	*		4.039	1.183	.281
						(.2084)			(1.639)	(.223)	(.074)
	1931	45	.4325	*	.6386	.9393	*		5.956	1.357	.214
						(.1641)			(1.927)	(.282)	(.077)
	1927	44	.1267	**	.5577	.5252	**	**X[a]	4.817	1.643	.328
						(.2127)			(1.363)	(.293)	(.102)
	1925	43	.3254	*	.5000	.6225	*	**X[a]	4.287	1.584	.345
						(.1400)			(1.280)	(.508)	(.083)

[a]X in the column entitled $\pi = 1$ denotes that π is greater than zero and less than one.

Table 19. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2098	1958	7	.9261	*	.1330	1.3713	*		9.752	4.168	.431
						(.1732)			(2.957)	(.915)	(.003)
	1954	9	.7049	*	.4258	.8314	*		7.634	3.536	.476
						(.2033)			(1.272)	(.577)	(.041)
	1947	7	.6558	**	.2916	1.2342	**		5.185	2.171	.435
						(.3998)			(1.494)	(.390)	(.004)
	1939	16	.8074	*	.3030	1.0110	*		2.178	1.075	.486
						(.1320)			(.600)	(.233)	(.055)
	1937	16	.6765	*	.3217	1.6526	*		2.180	1.004	.442
						(.3054)			(.757)	(.157)	(.101)
	1935	17	.6350	*	.2677	1.8127	*	**Y[b]	2.013	1.027	.460
						(.3548)			(.696)	(.155)	(.143)
	1931	18	.0623		.5041	.5639			3.131	.869	.281
						(.5467)			(1.268)	(.140)	(.095)
	1927	17	.4275	*	.3144	1.1348	*		2.666	1.227	.400
						(.3391)			(.762)	(.195)	(.100)
2099A	1947	12	.0722		.4323	.4639			3.503	1.678	.514
						(.5259)			(.778)	(.198)	(.100)
2099	1958	33	.1706	**	.6433	.7437	**		13.051	4.022	.282
						(.2945)			(4.993)	(.780)	(.099)
	1954	37	.2718	*	.3247	1.2562	*		10.001	3.177	.311
						(.3476)			(6.108)	(.553)	(.120)
	1947	32	.3954	*	.2230	1.4783	*		6.278	2.372	.345
						(.3338)			(2.312)	(.369)	(.131)

[b]Y in the column entitled $\pi = 1$ denotes that π is greater than one.

Table 19. (Continued)

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
2099	1939	31	.7199	*	.4023	1.2698	*		2.793	1.047	.351
						(.1471)			(1.266)	(.267)	(.094)
	1937	31	.4757	*	.4653	1.1592	*		3.128	.986	.295
						(.2260)			(1.800)	(.278)	(.131)
	1933	28	.2923	*	.5559	1.1774	*		3.302	.870	.234
						(.3593)			(1.654)	(.174)	(.102)
	1931	31	.1306	**	.6865	.7177	**		5.111	.927	.159
						(.3439)			(3.010)	(.211)	(.087)
	1927	23	.5089	*	.4129	1.1353	*		3.438	1.235	.279
						(.2434)			(1.435)	(.261)	(.103)

Table 20. Statistical results of Equation 72

Industry number	Year	N	r^2	F	log a	b (s_b)	$\pi = 0$	$\pi = 1$	V/L ($s_{V/L}$)	W/L ($s_{W/L}$)	W/V ($s_{W/V}$)
3000	1931	7	.7217	**	.7034	1.4383	**		1.532	.404	.269
						(.3994)			(1.260)	(.200)	(.143)
	1927	7	.8875	*	.3896	1.2824	*		1.680	.725	.433
						(.2042)			(1.015)	(.380)	(.118)
	1925	5	.1150		.0671	.5125			.995	.656	.575
						(.8208)			(.426)	(.184)	(.383)
3001	1931	37	.4151	*	.8378	1.1435	*		7.703	1.059	.128
						(.2294)			(3.213)	(.192)	(.040)
	1927	35	.1860	*	.6966	.6541	*		6.806	1.533	.218
						(.2382)			(2.080)	(.341)	(.006)
3002	1939	3	.8597		.5030	.9070			4.395	1.417	.300
						(.3663)			(1.774)	(.529)	(.055)
	1925	3	.4828		.2460	1.9998			4.098	1.477	.318
						(2.0699)			(2.127)	(.236)	(.139)
3003	1939	9	.4727	**	.3929	.7516	**		2.737	1.126	.379
						(.3000)			(.776)	(.281)	(.086)

APPENDIX C

Table 21. Estimates of technological change for Industry 201

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	492,125	990,664	203,813	294,456	2.414	.282	.117
1947	1,280,692	1,822,013	274,441	781,940	4.666	.322	.069
1949	1,508,678	1,962,391	284,145	926,085	5.310	.022	.004
1950	1,505,554	2,234,428	282,331	903,867	5.332	.352	.066
1951	1,688,174	2,338,336	297,010	1,055,920	5.684	.420	.074
1953	1,947,917	2,314,205	298,611	1,172,191	6.523	- .321	- .049
1954	1,930,974	2,340,700	311,366	1,234,617	6.202	1.069	.172
1955	2,284,920	2,496,523	314,266	1,328,671	7.271	.525	.072
1956	2,525,488	2,624,319	323,944	1,439,143	7.796	.264	.034
1957	2,523,707	2,546,387	313,106	1,453,190	8.060	- .044	- .005
1958	2,499,233	2,697,675	311,758	1,415,624	8.016	.372	.046
1959	2,627,338	2,873,161	313,225	1,521,667	8.388	.291	.035
1960	2,663,375	2,696,169	306,867	1,560,750	8.679	.366	.042
1961	2,777,435	2,984,879	307,072	1,595,471	9.045		

Table 21. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	4.861	.222	.046	.598	.402	.018	.099	1.000	4.41
1947	6.639	.134	.020	.610	.390	.008	.061	1.792	7.15
1949	6.906	1.008	.146	.614	.386	.056	- .052	2.011	3.78
1950	7.914	- .041	- .005	.600	.400	- .002	.068	1.906	3.64
1951	7.873	- .062	- .008	.625	.375	- .003	.077	2.036	3.49
1953	7.750	- .232	- .030	.602	.398	- .012	- .037	2.350	4.43
1954	7.518	.426	.057	.639	.361	.020	.152	2.263	2.60
1955	7.944	.157	.020	.581	.419	.008	.064	2.607	5.13
1956	8.101	.032	.004	.570	.430	.002	.032	2.774	4.16
1957	8.133	.520	.064	.576	.424	.027	- .032	2.863	2.90
1958	8.653	.520	.060	.586	.414	.025	.021	2.771	2.91
1959	9.173	.387	.042	.579	.421	.018	.017	2.829	4.13
1960	8.786	.934	.106	.586	.414	.044	- .002	2.877	3.08
1961	9.720			.574	.426			2.871	2.91

Table 22. Estimates of technological change for Industry 202

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	283,537	587,779	86,426	110,040	3.281	.392	.119
1947	595,168	1,212,281	92,693	225,312	6.421	.355	.055
1949	689,145	1,355,093	96,642	263,971	7.131	- .587	- .082
1950	685,742	1,603,808	104,797	278,343	6.544	.798	.122
1951	730,517	1,787,238	99,498	293,406	7.342	.618	.084
1953	843,958	1,614,734	98,380	340,843	8.578	- .091	- .011
1954	781,098	1,654,911	92,034	319,246	8.487	- .289	- .034
1955	2,411,931	1,844,112	294,221	1,156,137	8.198	.158	.019
1956	2,504,747	2,076,705	299,772	1,211,092	8.356	.662	.079
1957	2,676,200	1,909,916	296,751	1,264,747	9.018	.740	.082
1958	2,866,779	3,275,812	293,802	1,346,026	9.758	.754	.077
1959	3,040,607	3,499,610	289,262	1,34[illegible],761	10.512	.522	.050
1960	3,164,914	3,598,849	286,842	1,404,274	11.034	.514	.046
1961	3,247,725	3,887,899	281,227	1,409,522	11.548		

Table 22. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	6.801	.785	.115	.388	.612	.070	.049	1.000	6.29
1947	13.078	.472	.036	.378	.622	.022	.033	1.392	8.26
1949	14.022	1.282	.091	.383	.617	.056	- .138	1.484	9.11
1950	15.304	2.658	.174	.406	.594	.103	.019	1.279	7.21
1951	17.962	- .774	- .043	.402	.598	- .026	.110	1.303	5.49
1953	16.413	1.569	.096	.404	.596	.057	- .068	1.590	5.58
1954	17.982	-11.714	- .651	.409	.591	- .385	.351	1.482	6.32
1955	6.268	.660	.105	.479	.521	.055	- .036	2.002	5.77
1956	6.928	- .492	- .071	.484	.516	- .037	.116	1.930	5.12
1957	6.436	4.714	.732	.472	.528	.386	- .304	2.154	5.34
1958	11.150	.948	.085	.470	.530	.045	.032	1.499	5.56
1959	12.098	.448	.037	.450	.550	.020	.030	1.547	5.30
1960	12.546	1.279	.102	.444	.556	.057	- .011	1.593	5.24
1961	13.825			.434	.566			1.575	5.10

Table 23. Estimates of technological change for Industry 203

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	317,496	643,162	151,417	117,993	2.097	.306	.146
1947	916,621	1,420,447	201,627	418,669	4.546	- .006	- .001
1949	928,069	1,590,845	204,745	481,921	4.533	.961	.212
1950	1,117,681	1,762,508	203,432	467,469	5.494	.004	.001
1951	1,185,795	1,923,813	215,693	560,717	5.498	.450	.082
1953	1,320,242	2,017,750	206,328	565,114	6.399	.132	.021
1954	1,301,195	2,069,584	199,238	573,057	6.531	.290	.044
1955	1,420,582	2,230,086	208,258	600,595	6.821	1.359	.199
1956	1,738,395	2,519,942	212,522	665,201	8.180	- .947	- .116
1957	1,543,005	2,530,453	213,319	655,513	7.233	1.256	.173
1958	1,895,705	2,668,750	223,323	741,856	8.489	.270	.032
1959	2,051,781	2,844,568	234,239	807,214	8.759	1.158	.132
1960	2,345,798	3,055,909	236,503	851,925	9.917	.274	.028
1961	2,476,062	3,002,758	242,974	900,408	10.191		

Table 23. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	4.248	.350	.082	.372	.628	.051	.095	1.000	7.44
1947	7.045	.362	.051	.457	.543	.028	- .029	1.760	7.19
1949	7.770	.894	.115	.519	.481	.055	.157	1.658	4.65
1950	8.664	.255	.029	.418	.582	.017	- .016	1.918	8.32
1951	8.919	.430	.048	.473	.527	.025	.057	1.887	4.08
1953	9.779	.608	.062	.428	.572	.035	- .014	2.102	4.28
1954	10.387	.321	.031	.440	.560	.017	.027	2.072	3.92
1955	10.708	1.149	.107	.423	.577	.062	.137	2.128	5.95
1956	11.857	.005	.0004	.383	.617	.0002	- .116	2.420	5.19
1957	11.862	.088	.007	.425	.575	.004	.169	2.139	3.73
1958	11.950	.194	.016	.391	.609	.010	.022	2.500	5.89
1959	12.144	.777	.064	.393	.607	.039	.093	2.555	6.16
1960	12.921	- .563	- .044	.363	.637	- .028	.056	2.793	6.03
1961	12.358			.364	.636			2.949	6.31

Table 24. Estimates of technological change for Industry 204

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	325,959	643,523	73,511	99,339	4.434	.552	.124
1947	1,001,692	1,340,812	113,217	311,736	8.848	- .176	- .020
1949	949,760	1,363,705	111,786	358,640	8.496	1.165	.137
1950	1,111,046	1,602,698	115,004	364,649	9.661	.725	.075
1951	1,169,396	1,667,859	112,598	393,905	10.386	.464	.045
1953	1,220,818	1,659,737	107,894	408,785	11.315	- .245	- .022
1954	1,217,055	1,773,907	109,944	434,858	11.070	1.044	.094
1955	1,344,068	1,871,257	110,950	452,026	12.114	.266	.022
1956	1,368,142	1,984,294	110,508	467,828	12.380	.877	.071
1957	1,423,514	2,053,884	107,375	473,559	13.257	2.339	.176
1958	1,855,693	2,521,770	118,984	575,775	15.596	.355	.023
1959	1,894,632	2,641,155	118,775	591,664	15.951	.858	.054
1961	2,074,527	2,930,561	117,429	633,723	17.666		

Table 24. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	8.754	.386	.044	.305	.695	.030	.094	1.000	6.06
1947	11.843	.178	.015	.311	.689	.010	- .030	1.752	11.60
1949	12.199	1.737	.142	.378	.622	.088	.049	1.647	7.30
1950	13.936	.876	.063	.328	.672	.042	.033	1.728	6.98
1951	14.812	.285	.019	.337	.663	.012	.033	1.785	5.55
1953	15.383	.752	.049	.335	.665	.032	- .054	1.903	6.27
1954	16.135	.731	.045	.357	.643	.029	.065	1.800	6.95
1955	16.866	1.090	.065	.336	.664	.043	- .021	1.917	5.89
1956	17.956	1.172	.065	.342	.658	.043	.028	1.877	5.80
1957	19.128	2.066	.108	.333	.667	.072	.104	1.930	6.27
1958	21.194	1.043	.049	.310	.690	.034	- .011	2.131	6.32
1959	22.237	1.360	.061	.312	.688	.042	.012	2.108	6.58
1961	24.956			.305	.695			2.159	5.58

Table 25. Estimates of technological change for Industry 205

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	762,340	641,194	283,521	372,725	2.689	.275	.102
1947	1,365,809	909,466	279,368	765,202	4.889	.260	.053
1949	1,580,185	1,030,768	292,073	924,134	5.410	.171	.032
1950	1,639,228	1,121,789	293,720	914,968	5.581	.338	.060
1951	1,810,106	1,164,242	305,823	1,016,303	5.919	.412	.070
1953	1,929,230	1,245,673	286,091	1,055,934	6.743	.049	.007
1954	1,977,188	1,247,932	291,100	1,124,426	6.792	.632	.093
1955	2,192,023	1,394,020	295,265	1,185,624	7.424	.295	.040
1956	2,344,890	1,387,154	303,782	1,253,717	7.719	.453	.058
1957	2,482,370	1,456,133	303,746	1,299,312	8.172	.571	.070
1958	2,634,310	1,511,243	301,296	1,333,630	8.743	.375	.043
1959	2,778,767	1,541,814	304,764	1,405,234	9.118	.365	.040
1960	2,902,136	1,636,721	306,033	1,468,655	9.483	.147	.016
1961	2,871,770	1,712,447	298,212	1,471,277	9.630		

Table 25. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	2.262	.124	.054	.489	.511	.028	.074	1.000	6.68
1947	3.255	.137	.042	.560	.440	.018	.035	1.592	10.36
1949	3.529	.290	.082	.585	.415	.034	- .002	1.703	10.94
1950	3.819	- .012	- .003	.558	.442	- .001	.061	1.700	9.34
1951	3.807	.274	.072	.561	.439	.032	.038	1.804	7.16
1953	4.354	- .067	- .015	.547	.453	- .007	.014	1.941	6.49
1954	4.287	.434	.101	.569	.431	.044	.049	1.968	6.36
1955	4.721	- .155	- .033	.541	.459	- .015	.055	2.064	7.29
1956	4.566	.228	.050	.535	.465	.023	.036	2.178	6.99
1957	4.794	.222	.046	.523	.477	.022	.048	2.256	7.15
1958	5.016	.043	.008	.506	.494	.004	.039	2.364	6.62
1959	5.059	.289	.057	.506	.494	.028	.012	2.456	6.52
1960	5.348	.394	.074	.506	.494	.036	- .020	2.485	6.32
1961	5.742			.512	.488			2.435	5.23

Table 26. Estimates of technological change for Industry 206

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	115,947	717,059	34,446	41,551	3.366	.404	.120
1947	233,869	951,897	35,423	91,291	6.602	- .220	- .033
1949	199,449	790,986	32,371	100,007	6.161	.175	.028
1950	219,067	942,997	34,573	106,839	6.336	.846	.134
1951	237,360	957,503	33,051	105,775	7.182	.144	.020
1953	240,240	977,448	32,165	121,050	7.469	.844	.113
1954	250,759	987,624	30,166	117,834	8.313	1.645	.197
1955	288,881	986,492	29,010	121,262	9.958	1.984	.199
1956	332,761	988,821	27,864	126,873	11.942	.919	.077
1957	363,973	1,001,055	28,300	134,110	12.861	-1.054	- .082
1958	337,058	1,026,917	28,548	142,567	11.807	.139	.012
1959	399,813	1,089,067	33,467	168,336	11.946	2.238	.187
1960	460,580	1,139,233	32,472	173,011	14.184	- .769	- .054
1961	435,638	1,071,905	32,474	176,971	13.415		

Table 26. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	20.817	.757	.036	.358	.642	.023	.097	1.000	3.40
1947	26.872	-1.218	- .045	.390	.610	- .027	- .006	1.776	6.17
1949	24.435	2.841	.116	.501	.499	.058	- .030	1.755	5.58
1950	27.276	1.694	.062	.488	.512	.032	.102	1.702	7.05
1951	28.970	.709	.024	.446	.554	.013	.007	1.876	5.93
1953	30.388	2.352	.077	.504	.496	.038	.075	1.902	3.84
1954	32.740	1.265	.039	.470	.530	.021	.176	2.045	3.99
1955	34.005	1.482	.044	.420	.580	.026	.173	2.405	4.14
1956	35.487	- .114	- .003	.381	.619	- .002	.079	2.821	5.32
1957	35.373	.599	.017	.368	.632	.011	- .093	3.044	5.56
1958	35.972	-3.430	- .095	.423	.577	- .055	.067	2.761	4.28
1959	32.542	2.542	.078	.421	.579	.045	.142	2.946	4.04
1960	35.084	-2.076	- .059	.376	.624	- .037	- .017	3.364	0.76
1961	33.008			.406	.594			3.307	4.71

Table 27. Estimates of technological change for Industry 207

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	205,738	271,706	73,635	81,591	2.794	.450	.161
1947	587,065	615,827	91,710	214,558	6.401	- .428	- .067
1949	511,043	605,595	92,180	246,888	5.544	.427	.077
1950	575,960	675,288	96,452	253,251	5.971	.573	.096
1951	533,807	706,915	81,567	238,464	6.544	.262	.040
1953	584,325	699,306	82,655	254,727	7.069	.344	.049
1954	596,211	684,347	80,425	254,988	7.413	.474	.064
1955	642,845	708,268	81,509	264,932	7.887	.312	.040
1956	671,963	718,076	81,952	279,977	8.199	.844	.103
1957	723,287	739,590	79,985	286,222	9.043	.319	.035
1958	749,066	763,018	80,010	299,030	9.362	.637	.068
1959	783,183	817,374	78,324	308,824	9.999	.572	.057
1960	832,243	898,631	78,729	322,749	10.571	.461	.044
1961	870,899	924,051	78,941	336,014	11.032		

Table 27. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	3.690	.378	.102	.396	.604	.062	.099	1.000	10.08
1947	6.715	- .072	- .011	.365	.635	- .007	- .060	1.792	16.55
1949	6.570	.431	.066	.483	.517	.034	.043	1.577	8.56
1950	7.001	1.666	.237	.439	.561	.133	- .037	1.645	9.12
1951	8.667	- .104	- .012	.446	.554	- .007	.047	1.584	5.06
1953	8.460	.049	.006	.435	.565	.003	.046	1.733	6.35
1954	8.509	.180	.021	.427	.573	.012	.052	1.813	6.52
1955	8.689	.073	.008	.412	.588	.005	.035	1.907	7.60
1956	8.762	.485	.055	.416	.584	.032	.071	1.974	8.67
1957	9.247	.289	.031	.395	.605	.019	.016	2.114	8.22
1958	9.536	.900	.094	.399	.601	.056	.012	2.148	7.76
1959	10.436	.978	.094	.394	.606	.057	.000	2.174	8.58
1960	11.414	.292	.026	.388	.612	.016	.028	2.174	8.91
1961	11.706			.386	.614			2.235	9.01

Table 28. Estimates of technological change for Industry 208

Year	Value added ($1000) Q	Capital ($1000) K	No. of employees L	Payroll ($1000) W	$\frac{Q}{L} = q$	$\frac{\Delta q}{\Delta t} = \dot{q}$	$\frac{\dot{q}}{q}$
1939	656,198	1,296,512	131,559	222,396	4.988	.519	.104
1947	1,850,914	2,778,180	202,574	598,654	9.137	.211	.023
1949	1,956,249	3,153,861	204,647	697,914	9.559	.016	.002
1950	2,018,569	3,407,284	210,821	707,531	9.575	.316	.033
1951	1,985,411	3,885,944	200,723	739,350	9.891	.606	.061
1953	2,377,489	3,890,174	214,142	891,147	11.102	-.069	-.006
1954	2,237,428	4,030,710	202,795	867,680	11.033	1.025	.093
1955	2,432,461	3,982,589	201,729	905,476	12.058	.532	.044
1956	2,575,099	3,907,418	204,530	952,358	12.590	.351	.028
1957	2,618,724	4,114,854	202,363	978,586	12.941	.811	.063
1958	2,835,661	4,717,171	206,197	1,041,366	13.752	.978	.071
1959	3,071,497	4,915,065	208,516	1,099,551	14.730	.412	.028
1960	3,197,914	5,007,041	211,193	1,138,551	15.142	.378	.025
1961	3,234,052	5,512,964	208,384	1,164,883	15.520		

Table 28. (Continued)

Year	$k = \frac{K}{L}$	$\dot{k} = \frac{\Delta k}{\Delta t}$	$\frac{\dot{k}}{k}$	$\frac{W}{Q}$	$1 - \frac{W}{Q} = w_K$	$w_K \frac{\dot{k}}{k}$	$\frac{\dot{q}}{q} - w_K \frac{\dot{k}}{k}$	A(t)	Z
1939	9.855	.482	.049	.339	.661	.032	.072	1.000	9.77
1947	13.714	.848	.062	.323	.677	.042	- .019	1.576	10.70
1949	15.411	.751	.049	.357	.643	.032	- .030	1.516	9.02
1950	16.162	3.198	.198	.350	.650	.129	- .096	1.471	8.38
1951	19.360	- .597	- .031	.372	.628	- .019	.080	1.330	6.20
1953	18.166	1.710	.094	.375	.625	.059	- .065	1.543	5.58
1954	19.876	- .134	- .007	.388	.612	- .004	.097	1.443	4.67
1955	19.742	- .638	- .032	.372	.628	- .020	.064	1.583	5.86
1956	19.104	1.230	.064	.370	.630	.040	- .012	1.684	5.26
1957	20.334	2.543	.125	.374	.626	.078	- .015	1.664	5.49
1958	22.877	.695	.030	.367	.633	.019	.052	1.639	5.33
1959	23.572	.136	.006	.358	.642	.004	.024	1.724	5.97
1960	23.708	2.748	.116	.356	.644	.075	- .050	1.765	5.62
1961	26.456			.360	.640			1.677	5.39

ACKNOWLEDGEMENTS

The author wishes to thank Dr. Lehman B. Fletcher, his Major Professor, for his guidance, interest and counsel throughout the length of this study. Dr. Fletcher, on several occasions, rekindled the author's enthusiasm when progress was slow and the burden of other responsibilities were heavy.

The author is also indebted to Dr. Raymond R. Beneke for his encouragement and interest in this study.